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An Improved PAPR Reduction in OFDM Using Discrete Cosine Transform and Partial Transmit Sequence

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Abstract: Peak-to-Average Power Ratio (PAPR) is one of the most important issues of OFDM System. There are many proposed techniques to reduce PAPR. One of the earlier and effective proposed methods was partial transmit sequence (PTS) technique. In this paper, a new technique that implements inverse discrete cosine transform (IDCT) along with PTS concept to get an improved PAPR reduction level were introduced. Simulations are performed with QAM modulation, 64 subcarriers and variety number of partitions and admitted angle phases. The comparisons between the proposed technique with OFDM, PTS, and IDCT Precoded OFDM with PTS were conducted and the simulation result shows that the proposed technique surpass all three techniques in term of PAPR reduction. Furthermore, the proposed technique can reduce the maximum OFDM PAPR level to 7dB.

Keywords: OFDM, PAPR, partial transmit sequence, discrete cosine transform, CCDF.

1. Introduction

Orthogonal frequency division multiplexing (OFDM) is a parallel and multicarrier transmission scheme. OFDM is adopted in many famous standards these days such as DSL, 802.11a, Wi-Max, LTE and others due to plurality of provided advantages in compare to others. Some of OFDM features include high spectral efficiency, high data transmission bitrate, immunity to inter symbol interference (ISI) and robustness against multipath fading. Although it is a great technology for high data transmission bitrate, OFDM suffers from high peak-to-average power ratio (PAPR). High PAPR is considered the main drawback of this technique. Despite all of the great provided features, OFDM has one main drawback that increases OFDM system complexity, cost, and power consumption which is the high peak-to-average power ratio.

Due to parallelism used in OFDM, the sum of all subcarriers signal will result in some very high peaks in the output signal. Those high peaks cannot be easily amplified linearly by the final power amplifier connected to the transmission line which will distort the OFDM signal and will introduce lot of issues as a result of this distortion [1, 2]. To solve this issue, although it is costly, a very advanced and complex power amplifier system that can maintain linearity is needed.

To reduce system complexity, costs and power inefficiency of OFDM based systems, PAPR should be reduced. There are lots of techniques proposed for this purpose. The proposed techniques for PAPR reduction can be considered distortion techniques that include clipping or distortion-less technique such as PTS, SLM, coding, tone reservation, and tone injection.

Discrete cosine transform has been utilized in some proposed methods for PAPR reduction. In this paper, we propose a new distortion-less technique that utilizes discrete cosine transform (DCT) in an innovative way to enhance the output of PTS even further [3, 4]. The proposed method was tested and compared with conventional PTS and "Grouped DCT pre-coded OFDM combined with PTS".

The paper hereinafter is organized into sections as follows. Section II discusses about PAPR. Section III presents the concept of PTS technique which will be utilized for the proposed technique. Section IV presents the technique called Grouped DCT pre-coded OFDM combined with PTS. In section V, the proposed technique is discussed. Then, section VI shows the comparison results between proposed technique, PTS, "GDCT precoded OFDM combined with PTS" and OFDM. Finally, section VII concludes on the discussions of this paper.

2. Peak-to-Average Power Ration (PAPR)

The OFDM output signal can be represented by using the inverse discrete Fourier transform (IDFT) as:

$$x(t) = \frac{1}{N}\sum_{k=0}^{N-1} X_k . e^{j2\pi k\Delta f t} \text{ , } 0 \le t \le T \qquad (1)$$

where N is the number of subcarriers, X_k is the input data symbol, Δf is the frequency difference between adjacent subcarriers, and $T = 1/\Delta f$ is the symbol of duration. OFDM continuous time signal can also be calculated using matrix multiplication like below:

$$x = F^{-1}.X \tag{2}$$

where x is OFDM time domain signal sequence, F^{-1} is inverse Fourier matrix of $N \times N$ size and X is input data sequence with size of $N \times 1$. The inverse Fourier matrix is defined by:

$$F^{-1}{}_{xy} = \frac{1}{N} e^{j2\pi x \Delta f y}\, ; x, y = 0,1,..,N-1 \tag{3}$$

where x and $y = t$ represent column and row index respectively. So, in general

$$x = IDFT(X). \tag{4}$$

The term PAPR is defined as the ratio of peak power to average power of signals. The PAPR value for OFDM signal is calculated using the equation below:

$$PAPR = \frac{Max|x_k|}{E[|x_k{}^2|]} \quad ; k = 0, \dots NL-1 \tag{5}$$

where E denotes average and L is the oversampling value.

PAPR value can be viewed in dB so that

$$PAPR(dB) = 10\ log_{10}(PAPR). \tag{6}$$

That maximum PAPR for OFDM signal can be obtained when the input data X equals to the maximum allowed value for all inputs. As OFDM input data is mostly complex, X should have the same value for all inputs to generate its maximum PAPR. Therefore, for OFDM signal:

$$PAPR(dB)_{max} = 10\ \ log_{10}(N). \tag{7}$$

There are many methods proposed to reduce PAPR. To evaluate those proposed techniques, Complementary Cumulative Distribution Function (CCDF) is used. CCDF is a probability distribution function used to describe how often the random variable is above a particular value and CCDF for PAPR tells us how often the PAPR of OFDM output signal of random inputs is above $PAPR_o$ value. The more data used for calculating CCDF the more its info is trusted. CCDF is calculated as follows:

$$CCDF = \Pr(PAPR > PAPR_0). \tag{8}$$

3. Partial Transmit Sequence

Partial transmit sequence (PTS) is a technique based on combining signal sub-blocks which are phase-shifted by constant phase factors [5]. The main idea of PTS is to partition the original data block into U pair wise disjoint sub-blocks X_u, $u = 1,2, \cdots U$ as shown in Figure 1. Each subcarrier in an OFDM symbol is represented in exactly one of those sub-blocks. Although each sub-block can include an arbitrary number of subcarriers, the carrier positions represented in other sub-blocks need to be set to zero.

PTS needs U IDFT operations for each data block, and the amount of PAPR reduction depends on the number of sub-blocks U and the number of allowed phase factors.

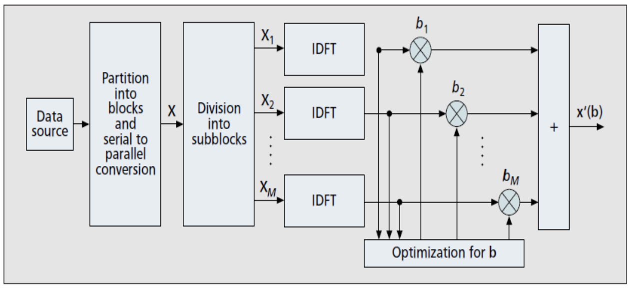

Figure 1. Block Diagram of PTS Technique [4].

The factor that may affect the PAPR reduction performance in PTS is the sub-block partitioning, which is the method of division of the subcarriers into multiple disjoint sub-blocks.

Therefore, there are two important issues that should be solved in PTS: high computational complexity for searching the optimal phase factors and the overhead of the optimal phase factors as side information needed to be transmitted to receiver for the correct decoding of the transmitted bit sequence [4].

Dividing subcarriers can be done in many ways. There are three partitioning schemes that can be used: adjacent, interleaved, and pseudo-random [5, 6]. After partitioning the OFDM symbols data sequence, each partition will be exposed to IDFT with inputs equal to the total number of subcarriers N_c. Then, the output of each IDFT block, which represents part of the total signal in time domain due to partitioning occurred in frequency domain, will be multiplied by a phase factor $b_m = e^{j\theta_m}$ so that the final output signal will be equal to

$$s = \sum_{m=1}^{M} x^{(m)}.b_m. \tag{9}$$

The rotation phase θ_m can be anything from 0 to 2π. Therefore to get the lowest possible PAPR for s signal, the system can simply try all possible combination of rotation phase values one by one to produce the vector $B = [b_1, b_2, \dots., b_M]$. It must be known that the receiver will need to know the phase factors prior to decoding OFDM signal, therefore the transmitter should send the used phase factor vector as side information. In practice, the number of side information bits should be kept within a reasonable limit by reducing the number of admitted rotation angles [5]. It is better to keep the admitted rotation angles between 2 and 4; therefore the maximum iteration will be W^M, where W is the number of admitted rotation angles.

Finally, and after finding the best phase factor values, the result of multiplication between each partial time sequence by its assigned optimal phase factor will be summed to produce the final signal shape with lower PAPR.

4. Grouped DCT Combined with PTS

The idea to use the DCT transform is to reduce the autocorrelation of the input sequence to reduce the peak to average power problem and it requires no side information to be transmitted to the receiver. Since the end of data for DCT is always continuous, the lower order of components will be dominated in the transform domain signal after converted by DCT. In this section, it will be briefly explained about the flow of this technique using DCT works.

The DCT/IDCT pair is given by,

$$X(n) = \frac{2}{N}\omega(n)\sum_{k=0}^{N-1} x(k)\frac{\pi n(2k+1)}{2N} \tag{10}$$

$with\ n = 0,1,\ldots,N-1$.

The inverse discrete cosine transform (IDCT) is,

$$x(k) = \sum_{k=0}^{N-1} \omega(n)X(n)\frac{\pi n(2k+1)}{2N} \tag{11}$$

$with\ n = 0,1,\ldots,N-1$ and $\omega(n) = \begin{cases} \frac{1}{\sqrt{2}} & ,n=0 \\ 1, & otherwise \end{cases}$

Let $t = \mathbf{n}\cdot\Delta\mathbf{t}, \mathbf{T} = \mathbf{N}\cdot\Delta\mathbf{t}$, where Δt is the sampling interval, the discrete time.

OFDM signal is,

$$s(n) = \sum_{k=0}^{N-1} A_k cos\frac{\pi n(2k)}{2N} \quad . \tag{12}$$

In order to use DCT, the DCT-OFDM signal is redefined as,

$$\boldsymbol{s(t) = \omega(t)\sum_{k=0}^{N-1} A_k cos\frac{\pi t(2k+1)}{2T}} \tag{13}$$

where

$$\boldsymbol{\omega(n)} = \begin{cases} \frac{1}{\sqrt{2}} & ,\mathbf{0} \le t \le \Delta t \\ \mathbf{1}, & \Delta t \le t \le T \end{cases}. \tag{14}$$

The discrete form is given by,

$$s(n) = \frac{2}{N}\omega(n)\sum_{k=0}^{N-1} A_k cos\frac{\pi n(2k+1)}{2N}. \tag{15}$$

It can be easily shown that due to the orthogonality between different subcarriers, the total power in an OFDM is the sum of the powers of all subcarriers P_i, where

$$P_i = \frac{1}{T}\int_0^T [A_i cos(\omega_i t + \emptyset_i)] = \frac{1}{2}A_i^2 \tag{16}$$

For the sake of enhancing the PAPR reduction of normal PTS technique, this scheme applies a discrete cosine transforms (DCT) as pre-coding to normal OFDM. The block diagram of this scheme is illustrated in Figure 2.

The signal is partitioned first before sending to DCT. The DCT pre-coding block multiplies each partition with DCT matrix so that for M partitions if we consider the input data $S = \sum_{m=0}^{M-1} S^{(m)}$ and in each partition sequence $S^{(m)} = [S_0^{(m)}, S_1^{(m)}, \ldots\ldots, S_{N-1}^{(m)}]^T$ then $X^{(m)} = C.S^{(m)}$ where C is the DCT matrix with size of $N \times N$.

$$C = \begin{bmatrix} C_{0,0} & \cdots & C_{0,N-1} \\ \vdots & \ddots & \vdots \\ C_{N-1,0} & \cdots & C_{N-1,N-1} \end{bmatrix} \tag{17}$$

The matrix is given by the equation below:

$$C_{ij} = \begin{cases} \frac{1}{\sqrt{N}} & , \quad i=0 \\ \sqrt{\frac{2}{N}}.\cos\left(\frac{(2j+1)i\pi}{2N}\right), & \end{cases} \tag{18}$$

where $i,j = 0,1,\ldots.,N-1$ and represent rows and columns index respectively.

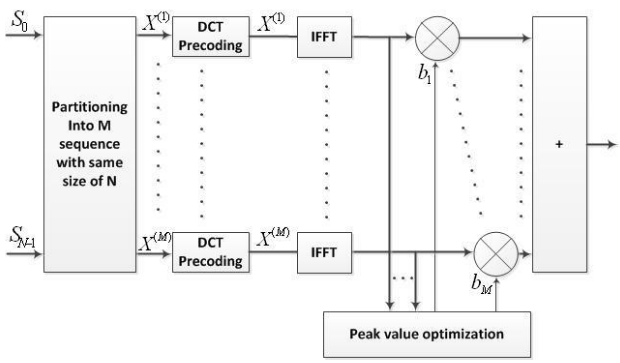

Figure 2. Implementation block diagram of GDCT pre-coded OFDM combined with PTS.

5. Proposed Method

The proposed method applies IDCT as IFFT pro-coding before PTS peak value optimization process. Figure 3 shows the block diagram of the proposed technique. If we consider the input data $X = [X_0, \ldots\ldots X_{N-1}]^T$, then the IDCT of X can be obtained using:

$$x(n) = \frac{1}{\sqrt{N}} X_0 + \sqrt{\tfrac{2}{N}} \sum_{k=0}^{N-1} X_k cos(\frac{\pi(2n+1)k}{2N}) \quad (19)$$

where $n = 0,2, \ldots., N-1$. The IDCT can also be achieved using matrix multiplication as below

$$x = IDCT(X) = C^{-1}.X$$

where C^{-1} is the $N \times N$ matrix of IDCT, and its value is calculated as below:

$$C^{-1}{}_{ij} = \begin{cases} \frac{1}{\sqrt{N}} & , \quad j = 0 \\ \sqrt{\frac{2}{N}}.\cos\left(\frac{(2i+1)j\pi}{2N}\right), & \end{cases} \quad (20)$$

where $i, j = 0,1, \ldots., N-1$ and represent rows and columns index respectively. The proposed technique introduce lower PAPR by optimum combination of partial transmit sequences as below:

$$s = \sum_{m=1}^{M} IDCT\left(IFFT\left(X^{(m)}\right)\right).b_m \quad (21)$$

where b_m is the optimum phase rotation value for m partition that results in total s signal with lowest PAPR.

At receiver side, the original data X can be retrieved by the equation:

$$X = FFT\big(DCT(s)\big).b^{-1} \quad (22)$$

where . refer to dot multiplication and b^{-1} is the inverse of b vector which is given by

$$b^{-1} = \frac{1}{b} \quad (23)$$

where b vector is $N \times 1$ size array and it is given by

$$b = \sum_{m=1}^{M} b^{(m)} \quad (24)$$

where

$$b^{(m)} = [b_0^{(m)}, b_1^{(m)}, \ldots., b_{N-1}^{(m)}]^T \quad (25)$$

And each array element is given by

$$b_k^{(m)} = \begin{cases} 1 & k \text{ is side information index} \\ b_m, & X_k^{(m)} \neq 0 \ ; k = 0,1,..,N-1 \\ 0, & otherwise \end{cases} \quad (26)$$

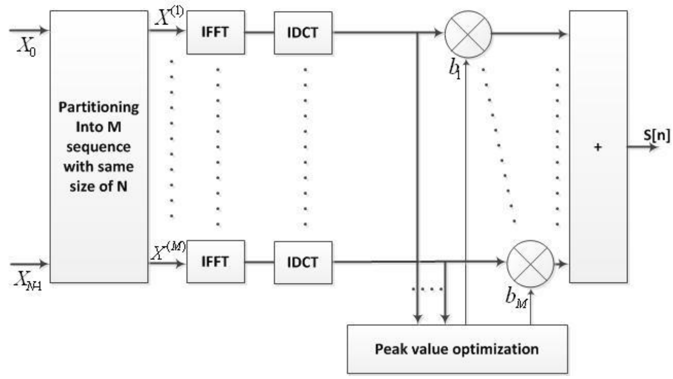

Figure 3. Implementation block diagram of the proposed technique.

6. Results and Discussion

The proposed method was tested for $N = 64$ and QAM4 for symbol mapping. The phase factor range used in simulation test was $\{\pm 1, \pm J\}$ for 2 and 4 partitions and $\{\pm 1\}$ for 2, 4 and 8 partitions. The CCDF was calculated for 100k frame of data to better analyze and evaluate the PAPR reduction performance of each scheme in the comparison. The PAPR is calculated for output signal generated using oversampling equal to 1.

This paper is intended to compare the proposed technique with PTS, to show how much enhancement is gained by the new improved PTS technique in this paper. The paper also compares the proposed method with GDCT Precoded OFDM, with PTS method as both here and GDCT Precoded OFDM and with PTS uses cosine transform along with PTS technique. The simulation considered only Adjacent and Interleaved partitioning schemes. As per [7], GDCT Pre-coded PTS deliver the best result with Adjacent partitioning. Adjacent is also the best partitioning scheme for conventional PTS as per [8, 9]. For the proposed method, the simulation results in Figure. 4 shows that Interleaved Partitioning is the best partitioning method.

The simulation results illustrated in Figure 5 and Figure 6 show that the proposed method enhances PTS PAPR reduction further. For 64 subcarriers and QAM, the CCDF of simulation comparison shows that the proposed method can reduce PAPR up to 5.6dB, 5.76dB, and 7dB for 2, 4, and 8 partitions respectively. The CCDF results also show that the result for M=4, W=2 is better than M=2, W=4 despite the fact that they are equal regarding the maximum iteration number $W^M = 16$. The CCDF plot also shows that the proposed method result surpass both PTS and GDCT Pre-code PTS method. Table 1 lists a full detail about all CCDF PAPR value at 10^{-5} for all configurations. The values, shown in the table, list the maximum PAPR value that an output signal of each technique could produce for probability of 99.99% of random inputs.

According to simulation results, the propose method can surpass PTS results by up to 2.39dB. In fact, the proposed method can reduce PAPR up to 7.05 dB compare to conventional OFDM. For OFDM with N=64, the maximum PAPR can be introduced when all input values are equal.

Table 1. A detailed list PAPR values for CCDF comparison at 10^{-5}.

M/W	OFDM	GDCT_PTS	New Method
2/4	11.94	8.12	6.32
4/2	11.94	7.47	6.20
4/4	11.78	7.30	4.91
8/2	11.94	6.33	4.89

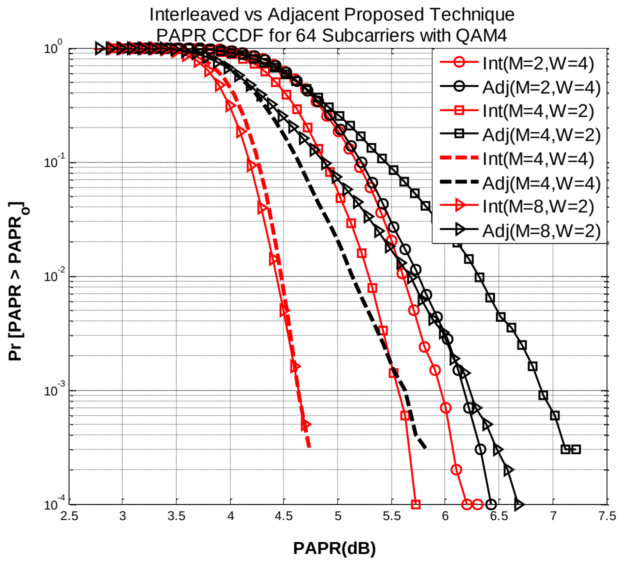

Figure 4. PAPR CCDF of proposed method implementation with both adjacent and interleaved partitioning for M=2, 4, 8 and W=2, 4 for comparison purpose.

7. Conclusion

A new method has been proposed to reduce OFDM PAPR. The proposed method utilize the concept of optimum combination of partial transmit sequence (PTS) and enhance its result by procoding fourier sub-blocks with inverse discrete cosine transform (IDCT). The proposed technique is examined using 64 subcarriers, QAM4 and various numbers of partitions and admitted phase angles. The simulation results showed a significant improving in term of PAPR reduction in comparison to OFDM and conventional PTS.

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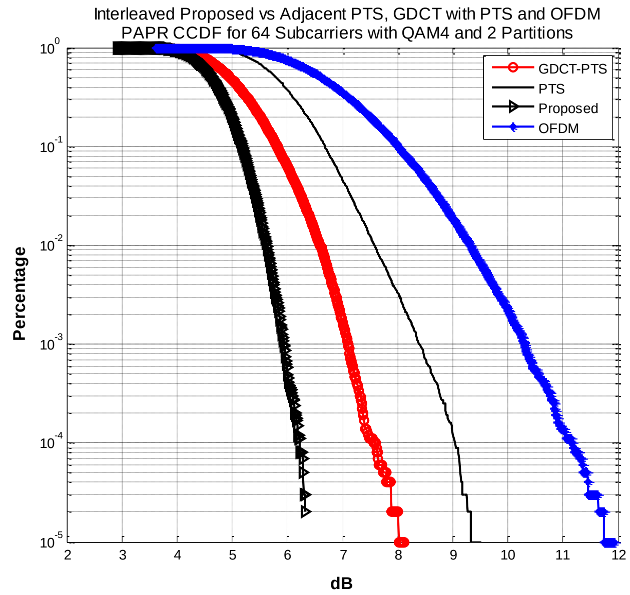

Figure 5. CCDF plot for proposed method (Interleaved), PTS, GDCT combined with PTS and OFDM implemented using QAM4, N=64, M=2, and W=4.

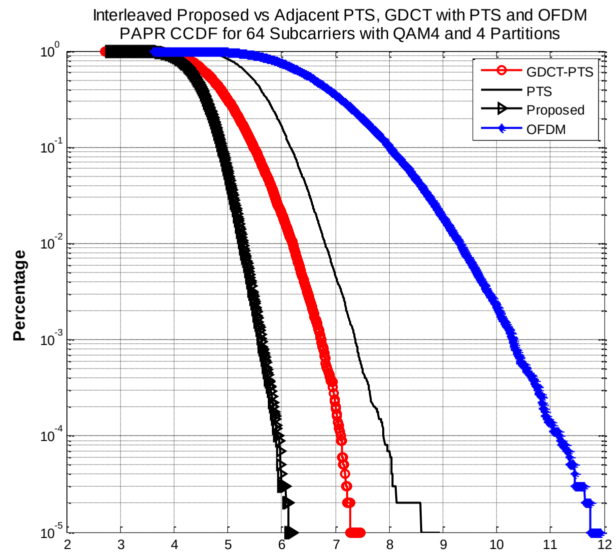

Figure 6. CCDF plot for proposed method (Interleaved), PTS (Adj), GDCT combined with PTS (Adj) and OFDM implemented using QAM4, N=64, M=4, and W=2.

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