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Analytical Solution of Squeezing Unsteady Nanofluid Flow in the Presence of Thermal Radiation

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Abstract: Homotopy perturbation method (HPM) and Runge-Kutta fourth-order method with shooting technique have been performed to investigate the presence of thermal radiation on squeezing unsteady nanofluid flow between two lateral plates. The results obtained, are found in good agreement with the previously published studies which employed Duan-Rach Approach (DRA). We found that the fluid temperature is physically decreased with an increase in thermal radiation parameter.

Keywords: nanofluid, squeezing flow, nonlinear equations, homotopy perturbation method, Runge-Kutta fourth-order method.

1. Introduction

The study of heat and mass transfer for squeezing unsteady viscous flow between two parallel plates is happening to be an amazing research topic as wide area of physical applications, such as lubrication system, polymer processing, food processing, hydrodynamical machines, compression, and crops damaging due to freezing, formation and dispersion amongst others. Sheikholeslami et al. [1] used heat line analysis to simulate two-phase simulation of nanofluid flow and heat transfer. Their results indicated that the average Nusselt number decreases with an increase of buoyancy ratio number until it reaches a minimum value and then starts increasing. As Lewis number increases, this minimum value occurs at higher buoyancy ratio number. Moreover, Sheikholeslami et al. [2] investigated the unsteady flow of a nanofluid squeezing between two parallel plates using Adomian decomposition method (ADM). Their results display that as the plates are moving together, they observed that as both the nanoparticle volume fraction and Eckert number are also increasing, so does the Nusselt number. Siddiqui et al. [3] investigated and analyzed the unsteady two-dimensional flow of a viscous magnetohydrodynamic MHD fluid between two parallel infinite plates. Their findings show that at a given time and for a specific figure of the constant parameter R the normal velocity rises significantly from the similarity variable $\eta = 0$ to $\eta = 1$, for different values of the magnetic parameter M. Hamza [4] considered the motion of an electrically conducting fluid film squeezed between two parallel disks in the presence of a magnetic field which applied perpendicular to the disks. He discovered that the electromagnetic force increased the load carrying capacity significantly. However, the problem of squeezing flow between rotating disks has been studied by Hamza [5] and later by Bhattacharyya [6]. He [7-10] proposed a new perturbation method which is indeed a reliance on the traditional perturbation method as Nayfeh [11], Krylovas [12], and the homotopy used in topology. This gives rise to the homotopy perturbation method (HPM).

Recently, Pourmehran et al. [13] analytically investigated the squeezing unsteady nanofluid flow between parallel plates by least square method (LSM) and collocation method (CM). Their results show that the Nusselt number increases with the increase of nanoparticle volume fraction and Eckert number while it decreases with the increase of the squeezing number. Nevertheless, Dib et al. [14] studied and approximated an analytical solution of squeezing unsteady nanofluid flow by using Duan-Rach approach (DRA). They obtained a good agreement with the numerical method by using fourth order Runge-Kutta algorithm. There are some researchers who studied the problem of squeezing fluid flow [15-21].

Radiation is a method of heat transfer that does not rely upon any contact between the heat source and the heated object as the case with conduction and convection. Heat can be transmitted through empty space by thermal radiation often called infrared radiation. Kandasamy et al. [22] studied the unsteady Hiemenz flow of Cu-nanofluid over a porous wedge in the presence of thermal stratification due to solar energy radiation using Lie group transformation. Many studies can be found in relation to the thermal radiation effect [23-26].

The aim of this study is to use HPM to derive the solution of squeezing unsteady Cu-nanofluid flow analytically. The thermal radiation effect on the temperature profile was analyzed. Meanwhile, the fourth-order Runge-Kutta method along with shooting technique was used for the verification of our results. All the salient parameters associated with this study were duly analyzed.

2. Problem description

In the present study, the flow and heat transfer of two-dimensional unsteady squeezing nanofluid through the lateral plates was observed as represented in Figure. 1. The distance between the two plates at any non-dimensional time t is given as $z = \pm l(1-\alpha t)^{\frac{1}{2}} = \pm h(t)$. For $\alpha > 0$, the two plates are squeezed until they touch each other at $= \frac{1}{\alpha}$, while for $\alpha < 0$, the two plates are separated. Here, α is a constant, l is the initial position (at $t = 0$) and z is the axial coordinate which is obviously considered as zero from the flow region, with the flow model as considered along x and y coordinates respectively. The variable t, is the non-dimensional time throughout the flow. The viscous dissipation effect and heat source as result of friction originated from the fluid flow shear are secured. Thus, this conduct transpire when the Eckert number is remarkably large. Meanwhile, the fluid is nanofluid which contained the copper material. The governing equations are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \qquad (2)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \qquad (3)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_P)_{nf}}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu_{nf}}{(\rho C_P)_{nf}}\left(4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right) - \frac{1}{\rho C_P}\frac{\partial q_r}{\partial y}, \qquad (4)$$

Here u and v represent the velocities in x and y directions, respectively. While p, T, T_∞, f, and ρ are the pressure, the fluid temperature, the surrounding temperature, the fluid, and the density respectively, ρ_{nf}, μ_{nf}, $(\rho C_P)_{nf}$ and k_{nf} are correspondingly the effective density, dynamic viscosity, heat capacity and thermal conductivity of the nanofluid as reported in [28]:

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_p,$$

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_p,$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \quad \text{(Brinkman)} \qquad (5)$$

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)} \text{(Maxwell-Garnett)},$$

subject to the following boundary conditions,

$$v = v_w = \frac{dh}{dt}, \quad T = T_H \text{ at } y = h(t),$$
$$v = \frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = 0 \text{ at } y = 0. \qquad (6)$$

The radiative heat flux in Eq. (4) is given by the Rosseland formula [29] as:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \qquad (7)$$

where σ^* and $k^*$are the Stefan-Boltzmann constant and the mean absorption number. Regarding some studies [30, 31], we presume that the temperature variation amidst the flow is significantly limited, and the expression T^4 may be considered as a linear function of temperature. Therefore, T^4 is expanded using Taylor series expansion about T_∞ and ignoring the higher-order terms, hence

$$T^4 = 4T_\infty^3 T - 3T_\infty^4. \qquad (8)$$

Substituting Eqs. (7) and (8) into Eq. (4) we obtain

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_P)_{nf}}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu_{nf}}{(\rho C_P)_{nf}}\left(4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right) + \frac{32\sigma^* T_\infty^3}{3\rho C_P k^*}\frac{\partial^2 T}{\partial y^2}, \qquad (9).$$

Introducing the following quantities,

$$\eta = \frac{y}{\left[l(1-\alpha t)^{\frac{1}{2}}\right]}, \quad u = \frac{ax}{[2(1-\alpha t)]}f'(\eta), \; v = -\frac{al}{[2(1-\alpha t)]}f(\eta),$$
$$\theta = \frac{T}{T_H}, \; A_1 = (1-\phi) + \phi\frac{\rho_s}{\rho_f}, \quad N = \frac{4\sigma^* T_\infty^3}{k_f \rho C_P k^*}, \qquad (10)$$

and using the non-dimensional variables in Eq. (10) into Eqs. (2), (3) and (9) and eliminating the pressure gradient by taking curl from the resulting equations (2) and (3) we obtain:

$$f^{iv} - SA_1(1-\phi)^{2.5}(\eta f''' + 3f'' + f'f'' - ff''') = 0 \; (11)$$

$$(12A_3 + 16A_2 N)\theta'' + 3PrSA_2(f\theta' - \eta\theta') + \frac{3PrEc}{(1-\phi)^{2.5}}\left(f''^2 + 4\delta^2 f'^2\right) = 0 \qquad (12)$$

where N is the thermal radiation given in Eq. (10), while A_2 and A_3 are dimensionless constants given by:

$$A_2 = (1-\phi) + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f}, \quad A_3 = \frac{k_{nf}}{k_f}. \qquad (13)$$

Now Eqs. (11) and (12) need to be solved subject to equation (14):

$$f(0) = 0, f''(0) = 0, \; f(1) = 1, f'(1) = 0, \theta'(0) = 0, \theta(1) = 1. \qquad (14)$$

Here S is the squeezing integer, Pr and Ec are the Prandtl and Eckert numbers, respectively. See Mustafa et al. [32]:

$$S = \frac{al}{2v_f}, \; Pr = \frac{\mu_f (\rho C_p)_f}{\rho_f k_f}, Ec = \frac{\rho_f}{(\rho C_p)_f}\left(\frac{ax}{2(1-\alpha t)}\right)^2, \delta = \frac{1}{x}. \qquad (15)$$

The following quantities are categorically used for practical interest as defined in Domairry and Hatami [33]

$$C_f = \frac{\mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=h(t)}}{\rho_{nf} v_w^2},\ Nu = \frac{-lk_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=h(t)}}{kT_H}. \quad (16)$$

It can be obtained from Eq. (10) that

$$C_f^* = \frac{l^2}{x^2(1-\alpha t)Re_x C_f} = A_1(1-\phi)^{2.5} f''(1),$$
$$Nu^* = \sqrt{1-\alpha t} Nu = -A_3\theta'(1). \quad (17)$$

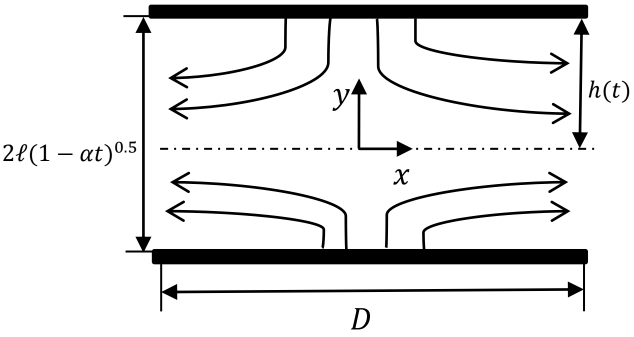

Figure 1. Geometry of the physical model.

2. Homotopy perturbation technique

To illuminate the fundamental concept of the technique, we take the general form of nonlinear differential equation into consideration as:

$$A(u) - g(r) = 0,\ r \in \Omega, \quad (18)$$

with boundary conditions

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0,\ r \in \Gamma, \quad (19)$$

In which A is the general differential operator, $u(r)$ is the fluid velocity, B is the boundary operator and $g(r)$ is the known analytic function, with Γ being the boundary of the domain Ω. A is the operator, which can be divided into both linear, L and nonlinear, $\boldsymbol{N}$, parts, respectively. Hence equation (18) could be defined in the form below:

$$L(u) + \boldsymbol{N}(u) - g(r) = 0. \quad (20)$$

Using the homotopy technique, the homotopy $v(r,p): \Omega \times [0,1] \to \mathbb{R}$ is constructed, and it suits the following expression:

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - g(r)], \quad (21)$$

where $p \in [0,1]$ is called an embedding parameter, while u_0 is the initial approximation of Eq. (18). Therefore, we can also write these as:

$$H(v,0) = L(v) - L(u_0),$$
$$H(v,1) = A(v) - g(r). \quad (22)$$

However, let consider the solution of Eq. (21) to be written as a power series in p:

$$v = v_0 + pv_1 + p^2 v_2 + \cdots, \quad (23)$$

where the final solution can be obtained when:

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + \cdots. \quad (24)$$

3. Solution with Homotopy Perturbation Method

Here, we apply the method to solve the velocity profile in the momentum equations (2) and (3). Then the value of the velocity obtained is used to solve the energy equation (4) to find the temperature distribution in the presence of thermal radiation.

According to Eq. (20), the differential equations (11) and (12) are decomposed into linear L and non-linear $\boldsymbol{N}$ operators. where $L(u) = f^{iv}$, $\boldsymbol{N}(u) = SA_1(1-\phi)^{2.5}(\eta f''' + 3f'' + f'f'' - ff''')$ and $g(r) = 0$. While $L(u) = (3A_3 + 4A_2 N)\theta''$ and $\boldsymbol{N}(u) = 3PrSA_2(f\theta' - \eta\theta') + \frac{3PrEc}{(1-\phi)^{2.5}}\left(f''^2 + 4\delta^2 f'^2\right)$ respectively.

We now apply HPM which satisfies Eq. (21) to nonlinear ordinary differential equation Eq. (11) as follows:

$$(1-p)[\ f^{iv}] + p[\ f^{iv} + SA_1(1-\phi)^{2.5}(\eta f''' + 3f'' + f'f'' - ff''')] = 0,\ p \in [0,1] \quad (25)$$

Suppose the solution of Eq. (25) has the form

$$f = f_0 + pf_1 + p^2 f_2 + \cdots. \quad (26)$$

Substituting Eq. (26) into Eq. (25) and using the boundary conditions (14), after some simplification and equating the like powers of p −terms, we obtain:

$$p^0:\ f_0^{iv} = 0, \quad (27)$$

$$f_0(0) = 0, f_0''(0) = 0,\ f_0(1) = 1, f_0'(1) = 0, \quad (28)$$

$$p^1:\ f_1^{iv} + SA_1(1-\phi)^{2.5}\begin{pmatrix}\eta f_0''' + 3f_0'' + f_0'f_0'' \\ -f_0 f_0'''\end{pmatrix} = 0 \quad (29)$$

$$f_1(0) = 0, f_1''(0) = 0,\ f_1(1) = 0, f_1'(1) = 0. \quad (30)$$

Solving Eqs. (27)-(30), we obtained the following approximations:

$$f_0(\eta) = \frac{1}{2}(3\eta - \eta^3), \quad (31)$$

$$f_1(\eta) = S\eta^5(1-\phi)^{2.5}A_1[0.00357143\eta^2 - 0.1]$$
$$+S\eta\sqrt{1-\phi}A_1[0.189286\eta^2 - 0.0928571$$

$$+0.185714\phi - 0.378571\eta^2\phi - 0.0928571\phi^2$$
$$+0.189286\eta^2\phi^2] \quad (32)$$

Hence, the required solution of Eq. (11), subject to Eq. (14) is obtained using the expression (24) as:

$$f(\eta) = \frac{1}{2}(3\eta - \eta^3) + S\eta^5(1-\phi)^{2.5}A_1[0.00357143\eta^2$$
$$-0.1] + S\eta\sqrt{1-\phi}A_1[0.189286\eta^2 - 0.0928571$$
$$+0.185714\phi - 0.378571\eta^2\phi - 0.0928571\phi^2$$
$$+0.189286\eta^2\phi^2]. \quad (33)$$

Similarly, having applied the same process in Eqs. (12) and (14), we obtained:

$$\theta_0(\eta) = 1, \quad (34)$$
$$\theta_1(\eta) = \frac{EcPr\sqrt{1-\phi}\eta^2}{(3A_3+4A_2N)(1-\phi)^5}[-2.25\eta^2 - 13.5\sigma^2$$
$$+4.5\eta^2\sigma^2 - 0.9\eta^4\sigma^2 + 4.5\eta^2\phi + 27\sigma^2\phi$$
$$-9\eta^2\sigma^2\phi + 1.8\eta^4\sigma^2\phi - 2.25\eta^2\phi^2$$
$$-13.5\sigma^2\phi^2 + 4.5\eta^2\sigma^2\phi^2 - 0.9\eta^4\sigma^2\phi^2]$$
$$+\frac{EcPr}{(3A_3+4A_2N)(1-\phi)^{2.5}}[2.25 + 9.9\sigma^2]. \quad (35)$$

Thus, the solution for the temperature profile is obtained as:

$$\theta(\eta) = 1 + \frac{EcPr\sqrt{1-\phi}\eta^2}{(3A_3+4A_2N)(1-\phi)^5}[-2.25\eta^2$$
$$-13.5\sigma^2 + 4.5\eta^2\sigma^2 - 0.9\eta^4\sigma^2 + 4.5\eta^2\phi$$
$$+27\sigma^2\phi - 9\eta^2\sigma^2\phi + 1.8\eta^4\sigma^2\phi - 2.25\eta^2\phi^2$$
$$-13.5\sigma^2\phi^2 + 4.5\eta^2\sigma^2\phi^2 - 0.9\eta^4\sigma^2\phi^2]$$
$$+\frac{EcPr}{(3A_3+4A_2N)(1-\phi)^{2.5}}[2.25 + 9.9\sigma^2] \quad (36)$$

Table 1. Thermophysical properties of water and nanoparticles [27].

Physical properties	ρ (kg/m³)	C_p (j/kg k)	k(W/mk)
Pure water	997.1	4179	0.613
Copper (Cu)	8933	385	401

4. Results and discussion

This study is much concerned about the development of the mathematical model of the unsteady nanofluid flow squeezed through the parallel plates as depicted in Figure 1, where the significance impact of the thermal radiation parameter along with other salient parameters on the flow and heat transfer features have been studied. Meanwhile, the missing parameters $f'(0), f'''(0)$ and $\theta(0)$ have been computed using the numerical integration. The present study is been validated by comparing with numerical study and previous studies reported in the literature [14] as in Figure 2 and Tables 2-4, respectively. The approximated values in Tables 2-4, show that the good agreement between the other two results and the present study on the velocity and the temperature profiles have been obtained.

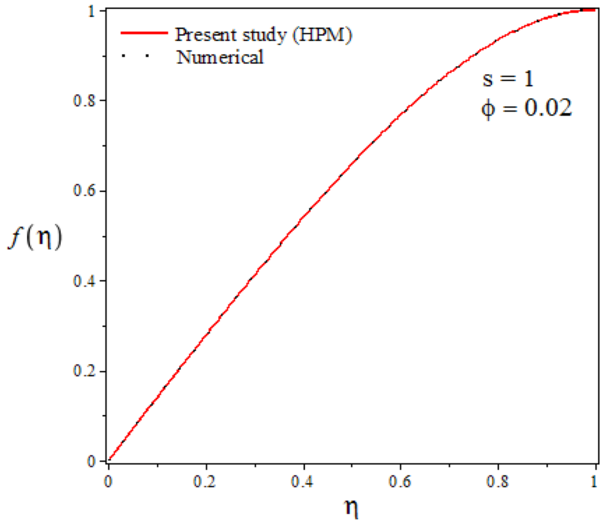

Figure 2. Results validation via HPM and numerical study.

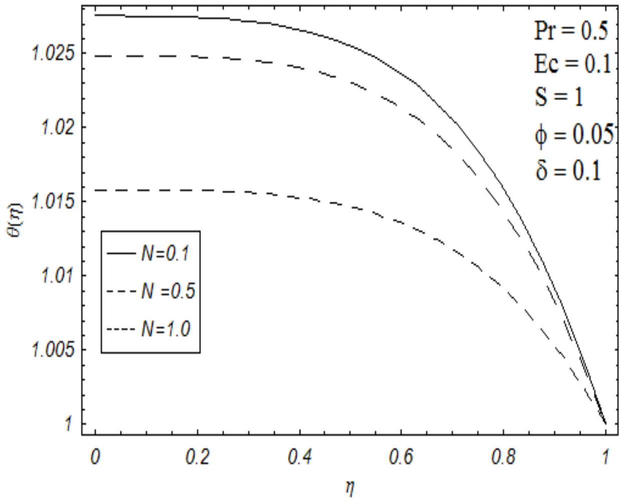

Figure 3. Effect of thermal radiation on the temperature profile (Cu-water).

Table 2. Comparison between the numerical and present study for $f(\eta)$ when $S = 1, Pr = 6.2, Ec = 0.01, \phi = 0.02$ (Cu-water), $\delta = 0.01$ and $N = 0$.

η	$f(\eta)$	
	Numerical	**Present study**
0	0.000000	0.000000
0.1	0.141359	0.141020
0.2	0.280666	0.280070
0.3	0.415781	0.415071
0.4	0.544379	0.543720
0.5	0.663857	0.663390
0.6	0.771229	0.771022
0.7	0.863016	0.863038
0.8	0.935120	0.935247
0.9	0.982695	0.982775
1	1	1

Table 3. Comparison between the numerical and present study for $\theta(\eta)$ when $S = 1$, $Pr = 6.2$, $Ec = 0.01$, $\phi = 0.02$ (Cu-water), $\delta = 0.01$ and $N = 0$.

η	$\boldsymbol{\theta(\eta)}$	
	Numerical	**Present study**
0	1.0320664	1.0328627
0.1	1.0320641	1.0328592
0.2	1.0320320	1.0328093
0.3	1.0318926	1.0325949
0.4	1.0315081	1.0320188
0.5	1.0306642	1.0308051
0.6	1.0290398	1.0285992
0.7	1.0261521	1.0249676
0.8	1.0212592	1.0193978
0.9	1.0131861	1.0112986
1	1	1

Table 4. Comparing of $-f''(1)$ between analytical results obtained by Dib et al. [14] and present study at different values of ϕ and S.

$\boldsymbol{\phi}$	$\boldsymbol{S}$	**Dib et al. [14]**	**Present study**
0.25	0.5	3.205131451	3.205131451
	1.0	3.410262902	3.410262902
	3.0	4.230788708	4.230788707
0.50	0.5	3.074439613	3.074439613
	1.0	3.148879226	3.148879225
	3.0	3.446637676	3.446637676

Table 5. Effect of N on $-\theta'(1)$ when $S = 1, Ec = 1, Pr = 0.5, \phi = 0.02$ and $\sigma = 0.01$.

N	0.1	0.5	1	5	50
$-\theta'(1)$	1.2979	0.9031	0.6543	0.2042	0.0234

Table 6. Effect of Pr and Ec on $-\theta'(1)$ when $S = 1, N = 2, \phi = 0.02$ and $\sigma = 0.01$.

Pr	0.5	1	1.5	3	1.5
Ec	1.5	1.5	1.5	1.5	0.5
$-\theta'(1)$	0.63282	1.26564	1.89847	3.79693	0.63282

Table 7. Effects of nanoparticles volume fraction and squeeze number on skin friction and Nusselt number, when $Ec = 1, Pr = 1.5, N = 2$ and $\sigma = 0.01$.

$\boldsymbol{\phi}$	$\boldsymbol{S}$	$\boldsymbol{f''(1)}$	$\boldsymbol{\theta'(1)}$	$\boldsymbol{C_{fx}}$	$\boldsymbol{Nu}$
0.0	0.0	-3.00000	-1.20331	3.53719	1.30311
0.01	0.5	-3.41065	-1.23393	3.92159	1.33627
0.03	1.0	-3.78044	-1.29852	4.13056	1.40621
0.05	3.0	-5.22249	-1.36794	5.41656	1.48139

Figure 3 and Table 5 show that the addition of thermal radiation effect in the model results in decreasing the fluid temperature. The fluid temperature physically decreases with an increase in thermal radiation parameter. Whereas, Table 6 presented the effect of both Eckert number and Prandtl number on the temperature profile, the temperature increases significantly with the increase of both Ec and Pr, respectively. Table 7 depicts the effects of nanoparticles volume fraction ϕ and squeeze number S on surface shear stress and dimensionless heat transfer rate with their corresponding physical quantities such as skin friction and Nusselt number, in which the shear stress and dimensionless heat transfer rate along with their physical quantities have all increase with an increase in both ϕ and S, respectively.

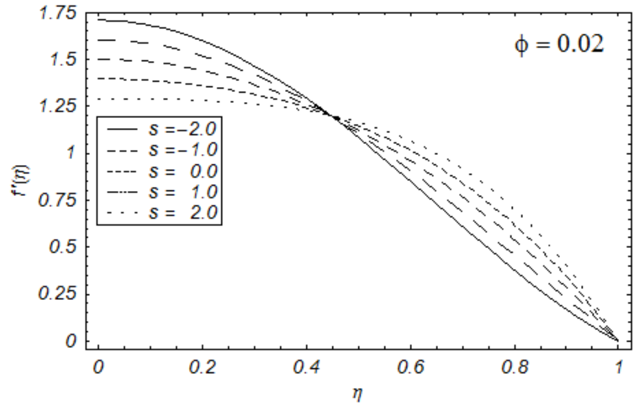

Figure 4. Effect of squeeze number of nanofluid on the velocity profile (Cu-water).

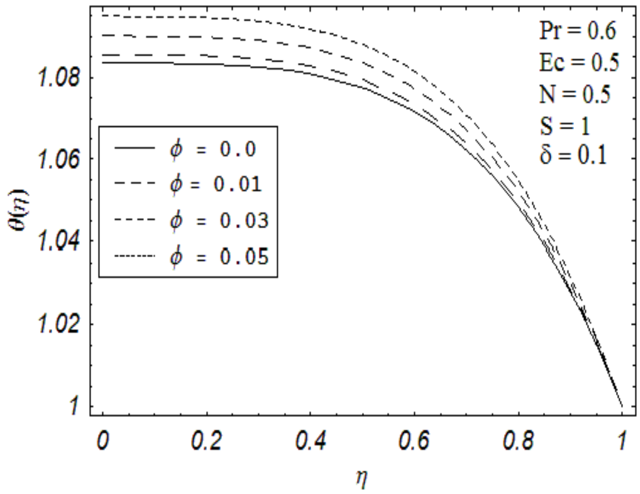

Figure 5. Effect of volume fraction of nanofluid on the temperature profile (Cu-water).

Figure 4 shows the effect of squeeze number on the velocity profile. The squeeze integer S, reports the motion of the plates, that is to say when $S > 0$ it coincides with the plates moving separately, while $S < 0$ corresponds with the plates moving collectively means the squeezing flow. Both positive and negative squeeze numbers have different effects on the velocity profile. The velocity increases with increase in the absolute value of squeeze number when $\eta < 0.5$, while it decreases for $\eta > 0.5$. Figure 5 displays that the volume fraction of nanofluid has an essential influence on the fluid temperature, as the temperature rises with an increment in the values of ϕ.

5. Conclusions

In this study, HPM was employed to obtain an approximate analytical solution of unsteady squeezing nanofluid flow between two lateral plates with thermal radiation being present. The fourth-order Runge-Kutta technique along with shooting scheme was used to verify the accuracy of our results. Thus, the desired and fascinate agreement between the analytical and numerical results, as well as the previously published study, were achieved. It has been found that the fluid temperature physically decreases with an increase in thermal radiation parameter. Additionally, the effects of the thermal radiation along with the other salient parameters involved have been discussed and presented in tables and graphs respectively. Ultimately, based on the auspicious and attractive features of the results produced by HPM, the method is highly recommended for its capability of handling many nonlinear problems in the field of science and engineering.

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