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The Linear GARCH Modelling of Nigerian Stock Prices

MohdTahir Ismail[1*], Buba Audu[1,2], Mohammed Musa Tumala[3], Edna Manga[1,2]

[1]School of Mathematical Sciences, UniversitiSains Malaysia,Pulau Pinang, Malaysia
[2]Department of Mathematics, Faculty of Natural Sciences, University of Jos, Nigeria
[3]Central Bank of Nigeria, Abuja, Nigeria
BubaAudu, School of Mathematical Sciences, UniversitiSains Malaysia,11800 Minden, Pulau Pinang, Malaysia
[*]Corresponding author email: mtahir@cs.usm.my

***Abstract*.** We used the monthly share prices of all Nigeria Share Price index from April, 2000 to January, 2014 to identify and model the volatility of asset return in the Nigerian Stock Exchange. We compared several ARMA-GARCH models that best fits the series. The result of our study shows that the ARMA(1,1)-GARCH (1,1) model best describes the volatility of the return. The volatility of the returns was found to be quite persistent, i.e. current volatility can be explained by past volatility that tends to persist over time.

***Keywords*: v**olatility, assetreturns, GARCH.

1. Introduction

The volatility modeling of price returns originated with [1] where autoregressive conditional heteroskedasticity models, ARCH model, was used to predict the uncertainty of UK inflation rate. Engle noted that large changes tend to be followed by large changes of either sign and small changes tend to be followed by small changes. This phenomenon was named volatility clustering. He measured the clustering effects through the assumption of a constant conditional mean of the returns.

However, there were some other stylized facts of volatility which could not be captured by the ARCH model. [2] a former student of Engle, generalized the ARCH model to the Generalized Conditionally Heteroskedasticity model (GARCH model). The model enormously extended the ability of the ARCH model to account for the stylized facts of volatility of returns.

Whereas many articles have been written about the volatility of stock markets, in Africa, and most especially Nigeria, little or not much has been done in this area due to lack of reliable source information. Therefore our focus in this study is to model and identify the volatility of asset returns in the Nigerian Stock Exchange Market.

The structure of this paper is as follows: In Section two we explain the materials and the methods used in the study. Furthermore, Section three discusses the data analysis and finally, Section four gives the main conclusion.

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2. Materials and Methods

Monthly Nigerian stock price index was collected from the data stream from April 2000 to January 2014. Thereafter the data generating process of the GARCH model is tested. To apply the GARCH model the variable is first examined for unit root and stationarity. The Augmented Dickey Fuller (ADF) test [3], [4] and Philip-Peron (PP) test [5] are used for this purpose. These preliminary tests are necessary in order to determine the order of non-stationarity of the data. If the series is integrated of order one, then using the GARCH model on the data results in better specification of the model. A good volatility model should be capable to capture and reflect the stylized facts about asset returns. Among the notable stylized facts are: volatility clustering, volatility persistence, volatility mean reversion, asymmetric effect, fat tails, influence of exogenous variable and co-movement of volatility. Details of these are found in [6]. In order to explore the returns volatility, the returns was calculated based on the following formulae:

$$r_t = \ln P_t - \ln P_{t-1} \tag{1}$$

where $P_t =$ the share price at period t and $P_{t-1} =$ the share price at period $t-1$ and r_t is the daily returns.

In practice, it is often found that a large number of lags p, and a large number of parameters are required to obtain a good model fit of ARCH (p) model. To circumvent this problem, [2] proposed the generalized GARCH(p,q) model with the following formulation:

$$\begin{aligned} h_t &= \alpha_0 + \sum_{i=1}^{p} \alpha_1 \varepsilon_{t-1}^2 + \sum_{i=1}^{q} \beta_i h_{t-i} \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2 + \beta_1 h_{t-1} + \cdots + \beta_q h_{t-q} \end{aligned} \tag{2}$$

where h_{t-1} is the volatility at day t-1, $\alpha_0 > 0, \alpha_i \geq 0$ $for\; i = 1, \cdots, p$ and $\beta_i \geq 0$ $for\; i = 1, \cdots, q.$

3. Analysis of Data

A time series plot of the returns series is presented in Fig. 1. A visual inspection of the plot reveals that some periods are riskier than others. Also the risky times are scattered randomly and there is some degree of autocorrelation in the riskiness of financial returns. The amplitudes of the returns vary over time as large (small) changes in returns are followed by large (small) changes. This phenomenon is called volatility clustering [6] and is one of the stylized facts of financial times. In addition, the plots depicted that as time went by, the shocks tend to persist, giving long memory process with high volatility half-lives in the asset returns. This phenomenon is called volatility persistence which is another stylized facts of the financial times [6].

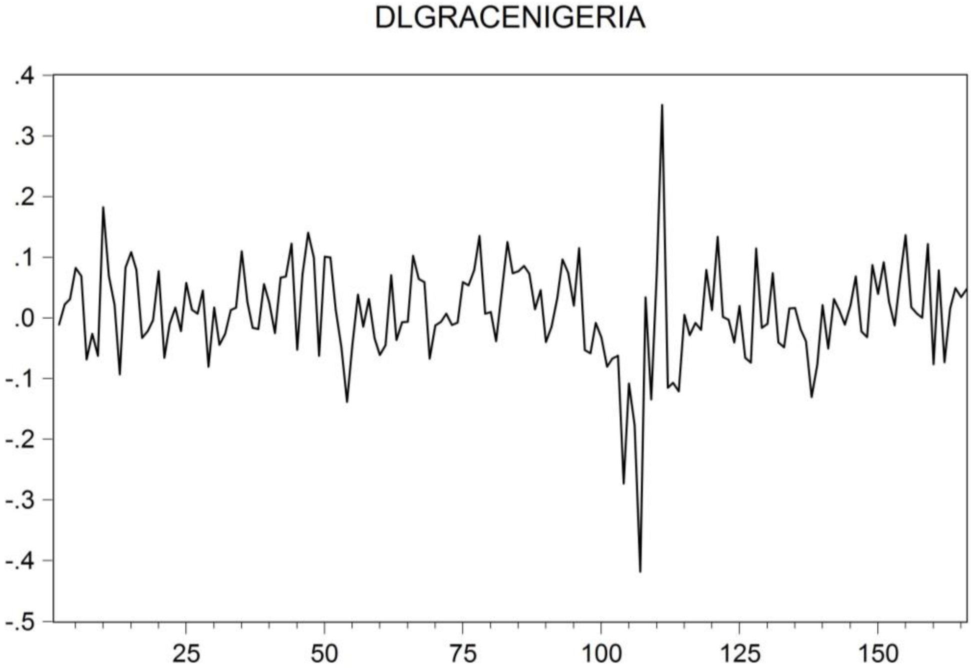

Figure 1. Time Plot of Nig. Monthly Stocks

We proceed to examine the autocorrelation functions of the log series and the returns series in Fig. 2 until Fig. 5. The autocorrelation function and the partial autocorrelation function show that the log series and the returns series are serially correlated. This indicates that there is a substantial dependence in the volatility of the log series and the returns series even though the returns series has no unit root.

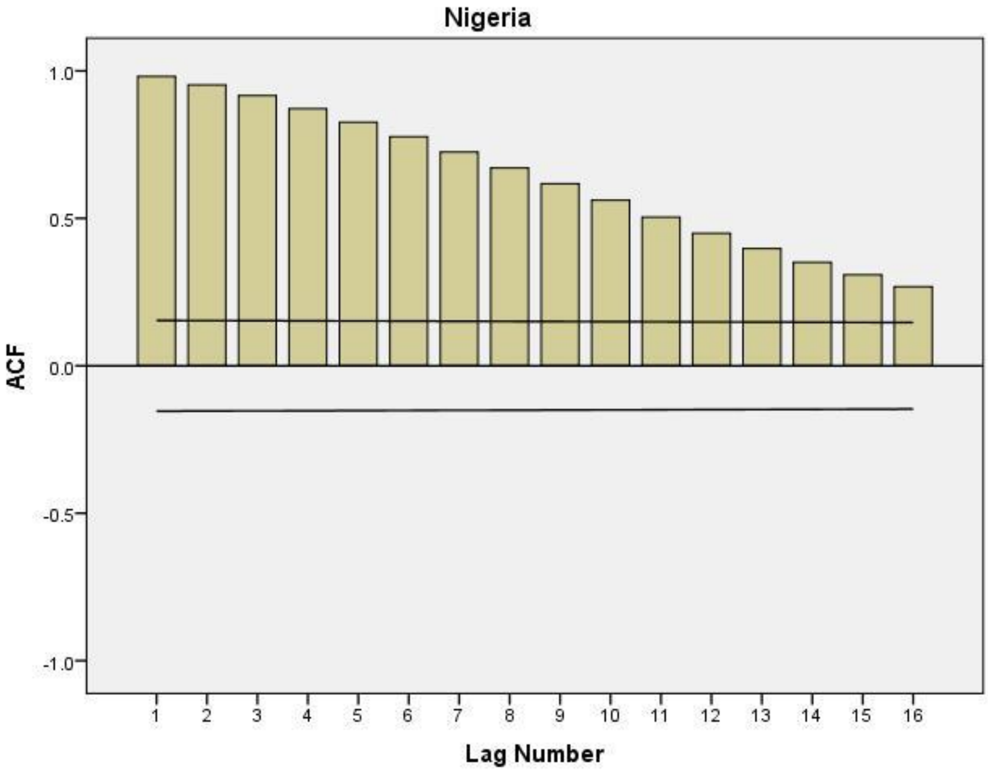

Figure 2. Nig. Stocks ACF.

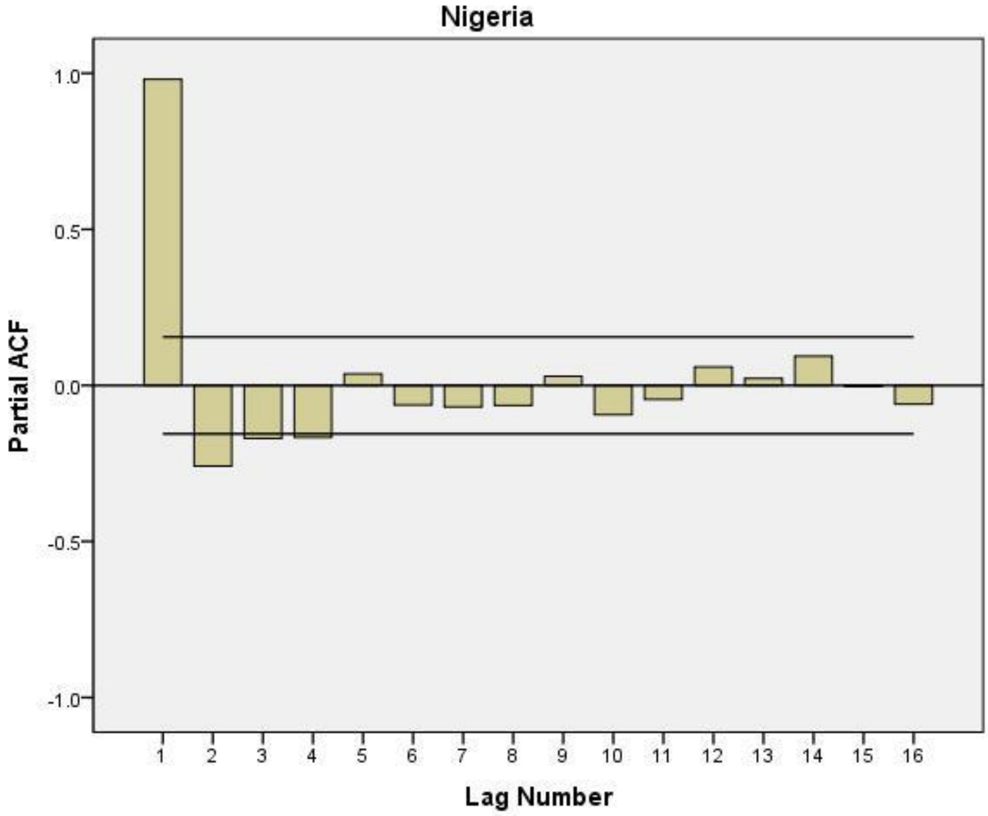

Figure 3. Nig. Stocks PACF

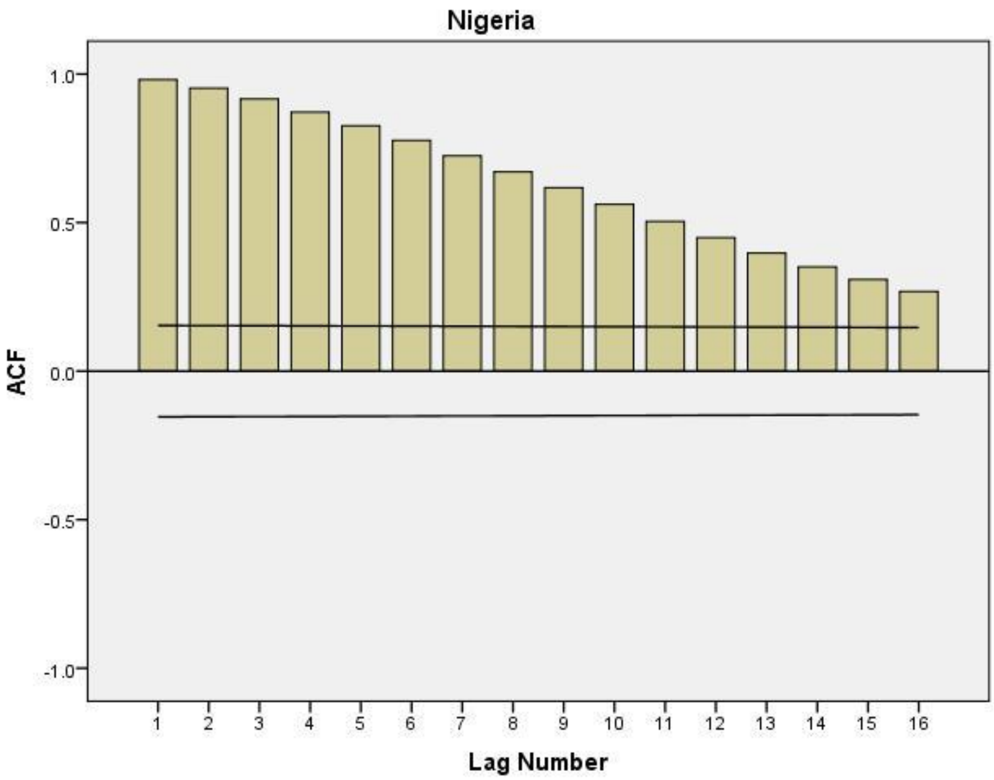

Figure 4. Nig. Stocks Returns ACF

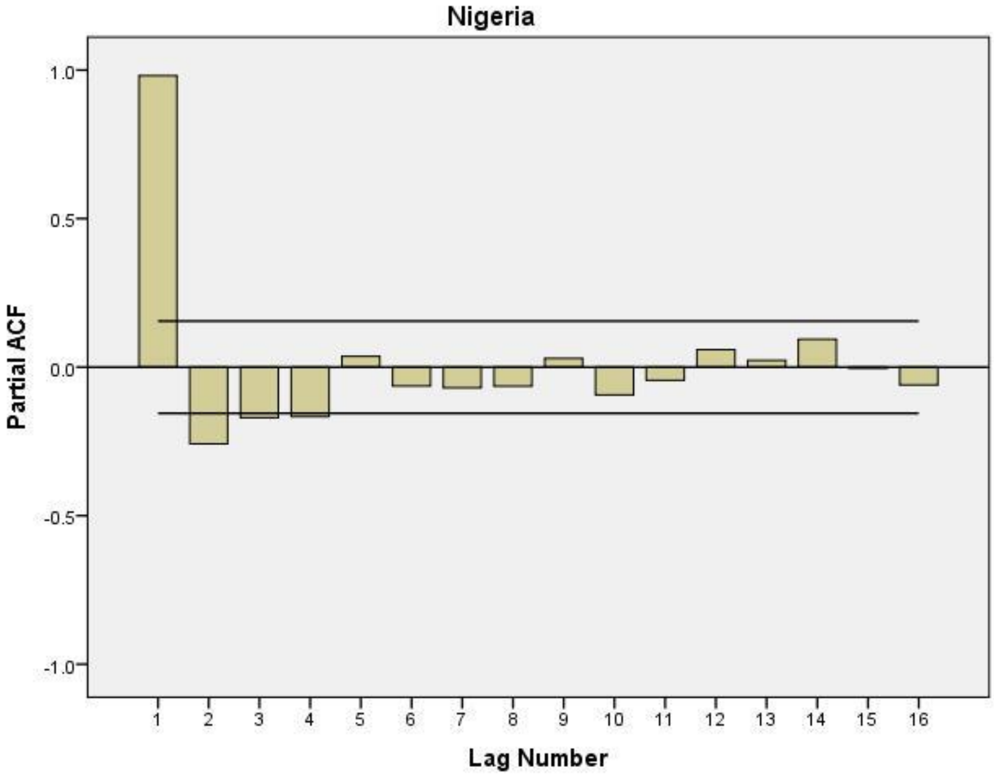

Figure 5. Nig. Stocks Returns PACF

Unit root test for the log series and the returns series data are conducted using the ADF test and the PP test at levels without constant and at levels with constant and unit root tests for the returns series is conducted at levels with none of either the constant withtrend or the constant withouttrend. The decision rule is to reject the null hypothesis when the value of the test statistic is less than the critical value.The results of the ADF and PP tests are shown in Table 1. The results indicate that the log series is not stationary at levels without constant and at levels with constant and trend while the returns series is stationary. Since the series is stationary after first differencing, it therefore means that the order of integration of the series is 1.

Table 1. Results of Unit Root Tests

	Augmented Dickey Fuller Test		Philips Perron Test	
	Intercept Without Trend	Intercept with Trend	Intercept Without Trend	Intercept with Trend
Level				
Log NMS	-2.4039 (0.1423)	-2.5048 (0.3255)	-1.875 (0.3434)	-1.8960 (0.6521)
First Difference [None]				
Log NMS	-4.3335*** (0.000)		-9.1149*** (0.000)	

Values in parenthesis are p-values. And *** indicates significance of the t-statistic at 1% p-value while ** indicates significance of the t-statistic at 5% p-value. * indicates significance of the t-statistic at 10% p-value.

In Table 2, the optimal lag lengths for both the AR and MA are selected using the information criteria. Out of these models a parsimonious model is obtained which has the lowest SC. The value of the SC for three candidate models are then computed and reported. We were guided by the ACF and PACF of the returns series in a choice of the three models.

Table 2. Results of Estimated SBC for The ARIMA Model

Model	ARIMA	SC
1	1,1,0	-2.214
2	0,1,1	-2.212
3	1,1,1	-2.198

Before the choice of our model, standard diagnostic checking of the residuals for ARCH effect is conducted on all the three candidate models. The results show that there is remaining ARCH effect in all the three candidate models. The results of the tests are presented in Table 3. The null hypothesis of no remaining ARCH-Effect in the residuals is rejected for all the three candidate models and therefore the conclusion that there is remaining ARCH-Effect is drowned on all the three candidate models. Hence, there is the need to model GARCH on the returns series.

Table 3. Results of Residual ARCH-Effect

ARIMA	F-Statistic	P-Value
1,1,0	7.589***	0.0065
0,1,1	6.295**	0.0131
1,1,1	5.870**	0.0166

*** indicates significance of the F-statistic at 1% P-value while ** indicates significance of the F-statistic at 5% P-value. * indicates significance of the F-statistic at 10% P-value.

The next thing we want to do is to estimate the parameters of different ARIMA-GARCH(1,1) model and choose a parsimonious model. The results are presented in Table: 4. The chosen model from the SC is an ARIMA(0,1,1)-GARCH(1,1).

Table 4. Results of Estimated SC for The ARIMA-GARCH(1,1) Model

Model	ARIMA-GARCH(1,1)	SC
1	1,1,0	-2.296
2	0,1,1	-2.310
3	1,1,1	-2.265
4	2,1,1	-2.226
5	1,1,2	-2.292
6	2,1,2	-2.250

In Table 5, we report the values of $(\alpha_1 + \beta_1)$ for all the six different models estimated in Table: 4. The values of $(\alpha_1 + \beta_1)$ are all positive and less than unity, but very close to unity indicating persistence. In addition, the values of the Z-Statistic are all significant.

Table 5. Results of Estimated $(\alpha_1 + \beta_1)$ for The ARIMA-GARCH(1,1) Model

ARIMA-GARCH(1,1)	$(\alpha_1 + \beta_1)$	Z-Statistic
1,1,0	0.80	3.571***
0,1,1	0.80	3.328***
1,1,1	0.81	3.570***
2,1,1	0.81	3.572***
1,1,2	0.83	3.565***
2,1,2	0.85	3.586***

*** indicates significance of the Z-statistic at 1% P-value while ** indicates significance of the Z-statistic at 5% P-value. * indicates significance of the Z-statistic at 10% P-value.

Figure: 6 and Figure 7 show the ACF and PACF of the residuals of the chosen ARIMA(0,1,1)-GARCH(1,1) model. The residuals ACF and PACF show that all the heteroscedasticity in the returns of the series has been fully accounted for as confirmed by the results of the formal test for the presence of ARCH-effect in table 3. It further confirms that the chosen model is good and adequate.

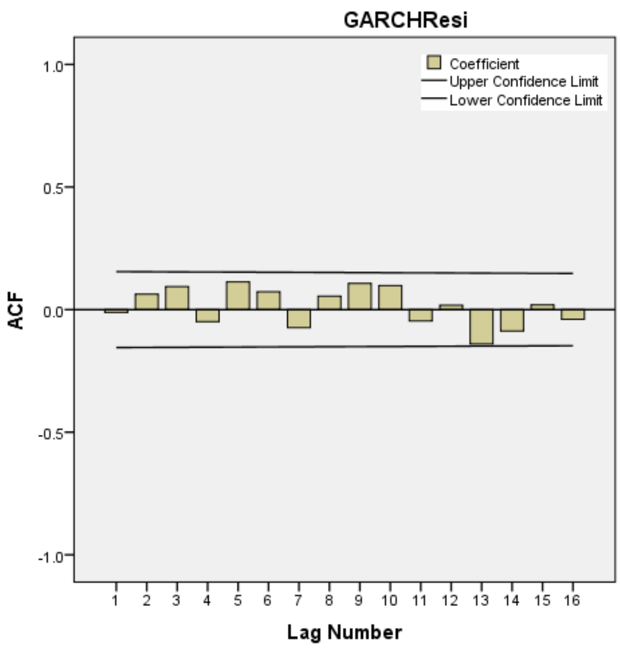

Figure 6. ARMA-GARCH Residuals ACF

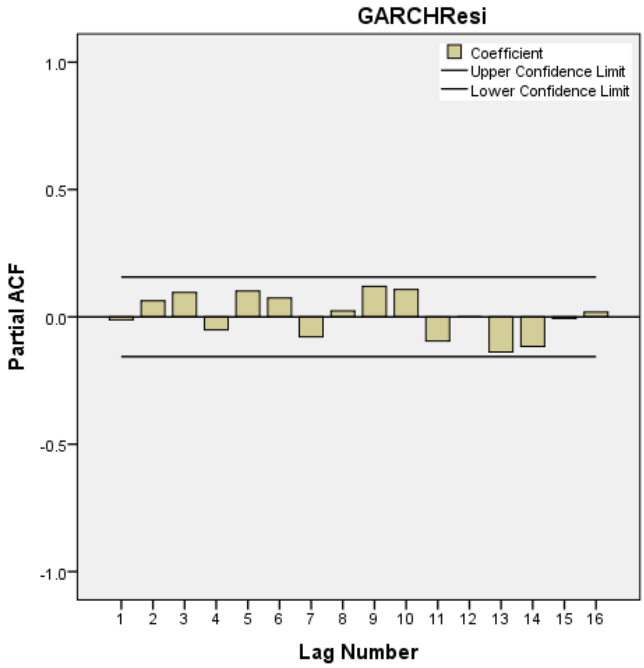

Figure 7.ARIMA-GARCH Residual PACF

4. Conclusion

An ARIMA(0,1,1)-GARCH(1,1) model is used to fit the Nigerian stock market returns. Our investigation shows that the volatility of the returns was found to be quite persistent i.e. current volatility can be explained by past volatility that tends to persist over time. The resultant model exhibit all other stylized facts such as mean-reverting behavior and fat tailed distribution.

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