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Solving Dynamic Equation by the Non-Standard Finite Difference Method

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Abstract: In this paper, we apply a non-standard finite difference method to find the numerical solution of dynamic nonlinear ordinary differential equation. The results of standard and non-standard finite difference schemes are also compared with each other. Some examples have been provided to show the ability of the method to show the adequacy of non-standard method, the results of two methods are compared with the exact solutions.

Keywords: Dynamic Equation, Non-Standard Method.

1. Introduction

Ronald Mickens developed numerical schemes using non-standard finite difference (NSFD) schemes for solving physical problems [1]. The method has been designed on the basis of the following steps:

Step 1: The first-order derivative must discrete in a more general form, i.e.

$$\frac{dy}{dx} \rightarrow \frac{y_{k+1} - \psi(h) y_k}{\phi(h)} \qquad (1)$$

where $\psi(h)$ and $\phi(h)$ are respectively called, the numerator and denominator functions, and have the following properties:

$$\psi(h) = 1 + o(h), \;\; \phi(h) = h + o(h^2) \qquad (2)$$

where $h = \Delta x,\ x \rightarrow x_k = hk,$ and $y(x) \rightarrow y_k$.

The second-order derivative is discretized as;

$$\frac{d^2 y}{dx^2} \rightarrow \frac{y_{k+1} - 2y_k + y_{k-1}}{\varphi(h)} \qquad (3)$$

where

$$\varphi(h) = h^2 + o(h^4) \quad . \qquad (4)$$

Step 2: Both linear and nonlinear terms involving the dependent variable may require "nonlocal" discretization; for example

$$y = 2y - y \rightarrow 2y_k - y_{k+1}$$
$$y^2 = yy \rightarrow y_k y_{k+1} \qquad (5)$$
$$y^3 = \frac{(y+y)}{2} y^2 \rightarrow \frac{(y_{k+1} + y_{k-1})}{2} y_k^2 \ .$$

The full details of these procedures are given by some studies [1-6]. The non-standard finite difference schemes have been developed as an alternative method for solving a wide range of problems such as the mathematical models of biology and chaotic systems [7-14].

2. Numerical results

In this section we solve dynamic nonlinear ordinary differential equation by NSFD method and compare the results of this method with those of standard finite difference (FD) method.

Consider the following general nonlinear first order dynamic equation with the given initial condition

$$y' = y(1 - y^n), \quad y(0) = 0.5 \qquad (6)$$

where n is a positive integer, when $n = 1$ equation (6) becomes;

$$y' = y(1 - y), \quad y(0) = 0.5 \ . \qquad (7)$$

This equation is called logistic differential equation. The exact solution of (7) is

$$y(x) = \frac{1}{1 + e^{-x}} \quad . \qquad (8)$$

We use the following NSFD scheme with nonlocal term for solving (7), i.e.

$$\frac{y_{k+1} - y_k}{(e^h - 1)} = y_k\,(1 - y_{k+1}) \ . \qquad (9)$$

Solving (9) for y_{k+1}, yields the following explicit NSFD scheme

$$y_{k+1} = \frac{e^h y_k}{1 + (e^h - 1) y_k} \quad . \qquad (10)$$

Note that, the nonlinear term y^2 in (7) was discrete as; $y_k y_{k+1}$. In figures 1 and 2 the non-standard scheme (10) is compared with the following standard scheme

$$\frac{y_{k+1} - y_k}{h} = y_k\,(1 - y_{k+1}) \qquad (11)$$

which yields

$$y_{k+1} = y_k + h y_k\,(1 - y_{k+1}) \quad . \qquad (12)$$

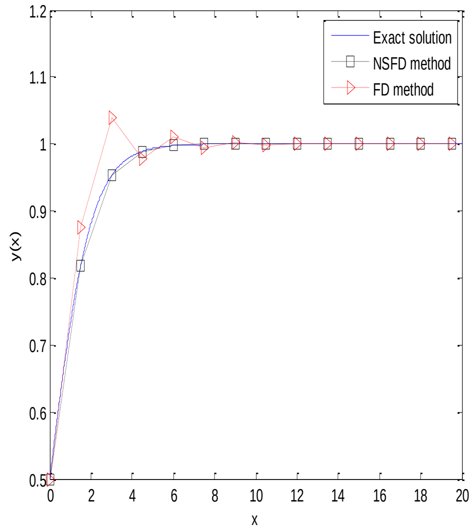

Figure 1. Numerical results of NSFD and FD methods for $h = 1.5$ and exact solution of equation (7)

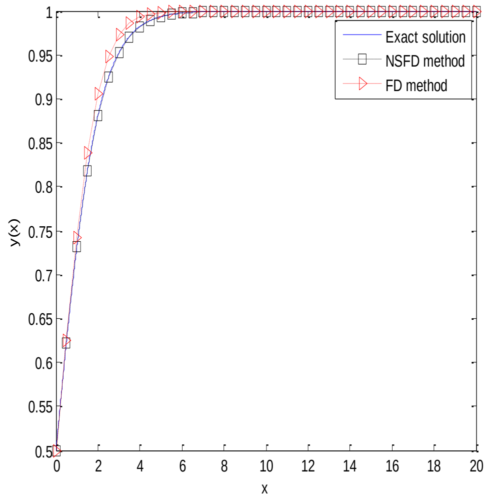

Figure 2. Numerical results of NSFD and FD methods for $h = 0.5$ and exact solution of equation (7)

In general we construct the following non-standard formula for (6)

$$\frac{y_{k+1} - y_k}{\left(\frac{1-e^{-nh}}{n}\right)} = y_k\,(1 - y_{k+1} y_k^{n-1}) \quad . \qquad (13)$$

Solving (13) for y_{k+1} yields the following explicit NSFD scheme

$$y_{k+1} = \frac{\left(n+1-e^{-nh}\right) y_k}{n + \left(1-e^{-nh}\right) y_k^n} \quad . \qquad (14)$$

For $n = 2$ we have the following scheme

$$y_{k+1} = \frac{\left(3-e^{-2h}\right) y_k}{2 + \left(1-e^{-2h}\right) y_k^2} \quad . \qquad (15)$$

In this case a standard scheme for solving dynamic equation is

$$\frac{y_{k+1} - y_k}{h} = y_k\,(1 - y_k^2) \qquad (16)$$

which yields

$$y_{k+1} = y_k + h y_k\,(1 - y_k^2) \quad . \qquad (17)$$

The exact solution of equation (6) for $n = 2$ is

$$y(x) = \frac{1}{\sqrt{1+3e^{-2x}}} \quad . \qquad (18)$$

In Figures 3 and 4 we compared the non-standard finite difference scheme (15) with the standard finite difference scheme (17).

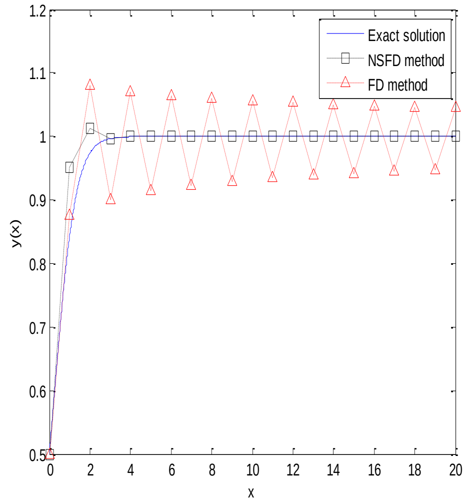

Figure 3. Numerical results of NSFD and FD methods for $h = 1$ and exact solution of equation (6) with $n = 2$

As shown in Figure 3, the standard scheme (17) for $h = 1$ is divergent while the non-standard scheme (15) is convergent to the exact solution (18).

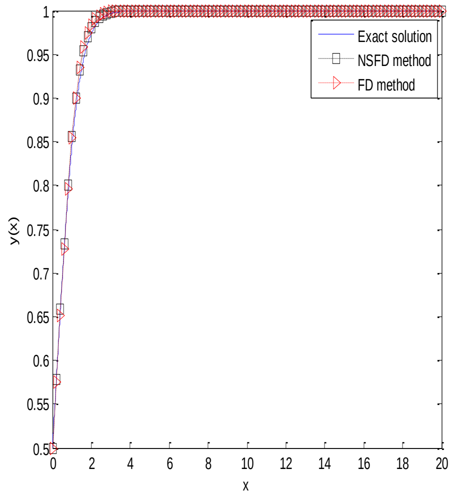

Figure 4. Numerical results of NSFD and FD methods for $h = 0.2$ and exact solution of equation (6) with $n = 2$

For $n = 3$ we have the following NSFD and FD schemes respectively

$$y_{k+1} = \frac{\left(4-e^{-3h}\right) y_k}{3 + \left(1-e^{-3h}\right) y_k^3} \qquad (19)$$

$$y_{k+1} = y_k + h y_k\,(1 - y_k^3) \quad . \qquad (20)$$

The exact solution of equation (6) for n =3 is

$$y(x) = \frac{1}{\sqrt[3]{1+7e^{-3x}}} \quad . \qquad (21)$$

In Figures 5, 6, 7 and 8 we compared the non-standard finite difference scheme (19) and standard finite difference scheme (20) for various values of h.

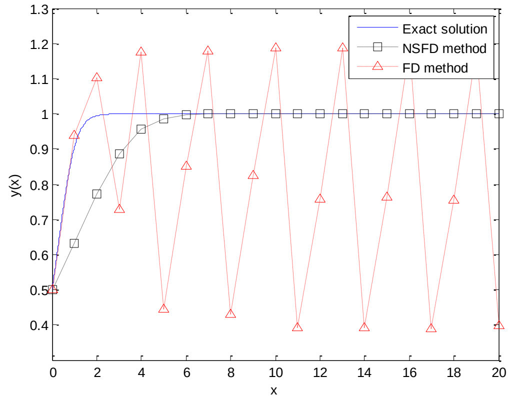

Figure 5. Numerical results of NSFD and FD methods for h =1 and exact solution of equation (6) with n =3

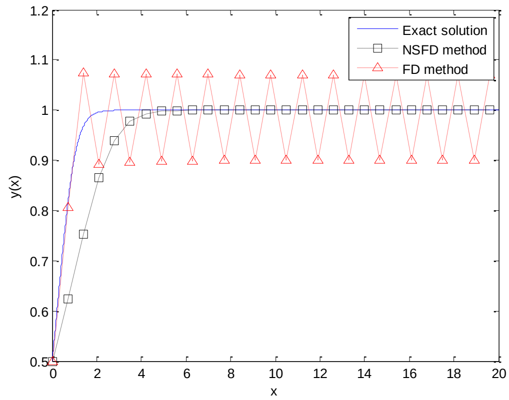

Figure 6. Numerical results of NSFD and FD methods for h = 0.7 and exact solution of equation (6) with n =3

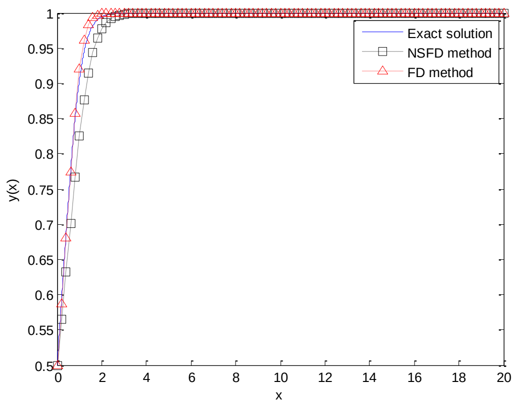

Figure 7. Numerical results of NSFD and FD methods for h = 0.2 and exact solution of equation (6) with n =3

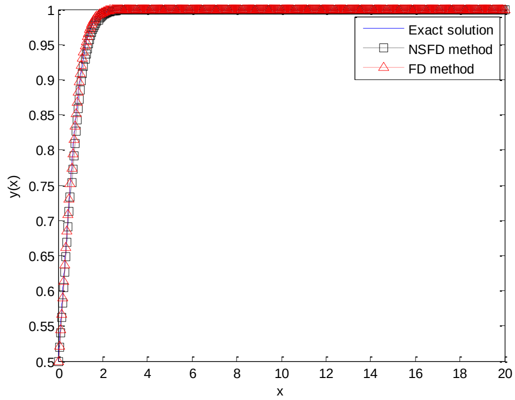

Figure 8. Numerical results of NSFD and FD methods for h = 0.05 and exact solution of equation (6) with n =3

For n = 20 we have the following NSFD and FD schemes respectively

$$y_{k+1} = \frac{\left(21 - e^{-20h}\right) y_k}{20 + \left(1 - e^{-20h}\right) y_k^{20}} \qquad (22)$$

$$y_{k+1} = y_k + h y_k \left(1 - y_k^{20}\right) \quad . \qquad (23)$$

The exact solution of equation (6) for n = 20 is

$$y(x) = \frac{1}{\sqrt[20]{1+(2^{20}-1)e^{-20x}}} \quad . \qquad (24)$$

In figures 9, 10 and 11 we compared the non-standard finite difference scheme (22) and standard finite difference scheme (23).

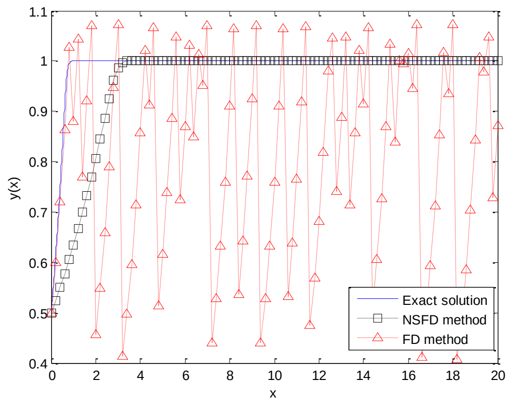

Figure 9. Numerical results of NSFD and FD methods for h = 0.2 and exact solution of equation (6) with n = 20

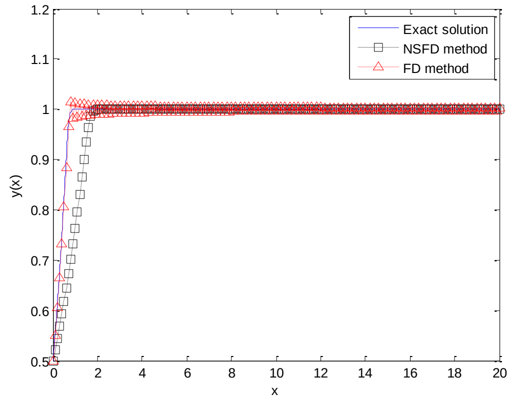

Figure 10. Numerical results of NSFD and FD methods for $h = 0.1$ and exact solution of equation (6) with $n = 20$

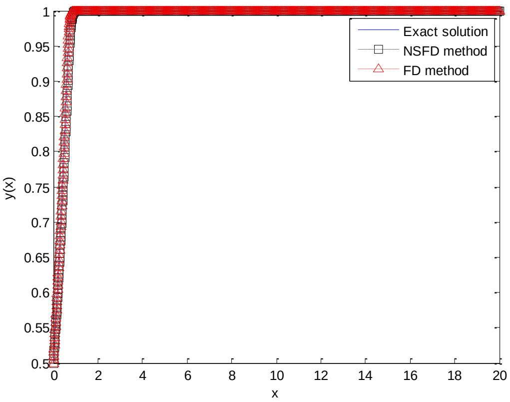

Figure 11. Numerical results of NSFD and FD methods for $h = 0.01$ and exact solution of equation (6) with $n = 20$

3. Conclusion

In this paper we have presented the efficiency of non-standard finite difference method in comparison with the standard finite difference method for numerical solution of dynamic equation. As figures of results show, the non-standard method is more stable than the standard method. It is also worth mentioning that the domain of h, for stability, in the non-standard method is larger than that of the standard method and non-standard method generates better results.

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