New Exact Traveling Wave Solutions for the Zakharov Equations and the Coupled Klein-Gordon-Zakharov Equations

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Abstract: The modified simplest equation method is an efficient method for obtaining exact solutions of some nonlinear partial differential equations. In the present work, the modified simplest equation method is used to construct exact solutions of the differential equations. In the present work, the modified simplest equation method is used to construct exact solutions of the Zakharov equations and the coupled Klein-Gordon-Zakharov equations. It also shown that the proposed method is effective and general.

Keywords: Modified simplest equation method, Zakharov equations, coupled Klein-Gordon-Zakharov equations.

1. Introduction

Nonlinear systems of partial differential equations (PDEs) play an important role in various areas of modern physics and engineering such as fluid mechanics, plasma physics, optical fibers and quantum mechanics. In theoretical investigation of the dynamics of strong Langmuir turbulence in plasma physics, various Zakharov equations take an important role [1, 2]. In this paper, we consider the following Zakharov equations

$$\begin{cases} n_{tt} - c_s^2 n_{xx} = \beta(|E|^2)_{xx}, \\ iE_t + \alpha E_{xx} = \delta n E, \end{cases}$$

and the coupled Klein-Gordon-Zakharov equations:

$$\begin{cases} u_{tt} - c_0^2 \nabla^2 u + f_0^2 u + \delta u v = 0, \\ v_{tt} - c_0^2 \nabla^2 v - \beta \nabla^2 |u|^2 = 0. \end{cases}$$

Zakharove equations and coupled Klein-Gordon-Zakharov equations are model systems to describe nonlinear interactions in plasma,where E or u denotes the electric field (more precisely it denotes its slowly varying envelope), n or v denotes the ion density fluctuation.

More recently, some exact solutions for the Zakharov and the coupled Klein-Gordon-Zakharov equations are obtained by using different methods [3-18]. In this work, we apply the modified simplest equation method to the Zakharov and the coupled Klein-Gordon-Zakharov equations. The modified simplest equation method is one ofthe most powerful and direct methods for constructing solutions of nonlinear partial differential equations.

In this paper, we have proposed the modified simplest equation method, and have presented applications for this method to nonlinear partial differential equations. The rest of the paper is organized as follows. In section 2, we describe the modified simplest equation method for finding traveling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent sections, we will apply the method to find exact traveling wave solutions of the Zakharov and the coupled Klein-Gordon-Zakharov equations. In the last section, some conclusions are presented.

2. Description of the modified simplest equation method

In this section, we briefly review the modified simplest equation method [19-23]. That is based on the assumption that the exact solutions can be expressed by a polynomial in $\frac{F'}{F}$ such that $F = F(\xi)$ is an unknown linear ordinary equation to be determined later. This method consists of the following steps:

Step 1. Consider a general form of nonlinear partial differential equation (PDE)

$$P(u, u_x, u_t, u_{xx}, u_{tx}, \dots) = 0 \qquad (1)$$

Assume that the solution is given by $u(x,t) = U(\xi)$ where $\xi = x + ct.$ Hence, we use the following changes:

$$\begin{aligned} \frac{\partial}{\partial t}(0) &= c\frac{\partial}{\partial \xi}(0), \\ \frac{\partial}{\partial x}(0) &= \frac{\partial}{\partial \xi}(0), \\ \frac{\partial^2}{\partial x^2}(0) &= \frac{\partial^2}{\partial \xi^2}(0) \end{aligned} \qquad (2)$$

and so on for other derivatives. Using (2) changes the PDE (1) to an ODE

$$Q(U, U'U'', \dots) = 0. \qquad (3)$$

where $U = U(\xi)$ is an unknown function, Q is a polynomial in the variable U and its derivatives.

Step 2.We suppose that Eq. (3) has the following formal solution:

$$U(\xi) = \sum_{i=0}^{N} A_i \left(\frac{F'}{F}\right)^i, \qquad (4)$$

where A_i are arbitrary constants to be determined such that $A_N \neq 0$ while $F(\xi)$ is an unknown function to be determined later.

Step 3. We determine the positive integer N in (4) by balancing the highest order derivatives and the nonlinear terms in Eq.(3).

Step 4. We substitute (4) into (3), we calculate all the necessary derivatives $U, U', U'', \dots$ and then we account the function $F(\xi)$. As a result of this substitution, we get a polynomial of $\frac{F'(\xi)}{F(\xi)}$ and its derivatives. In this polynomial, we equate all the coefficients to zero. This operation yields a system of equations which can be solved to find A_i and $F(\xi)$. Consequently, we can get the exact solution of Eq.(1).

3. Application the modified simplest equation method

In this section, we study the Zakharov and the coupled Klein-Gordon-Zakharov equation using the modified simplest equation method.

3.1 The modified simplest equation method to the Zakharov equations

Consider the Zakharov equations

$$\begin{cases} n_{tt} - c_s^2 n_{xx} = \beta(|E|^2)_{xx}, & (5) \\ iE_t + \alpha E_{xx} = \delta n E, & (6) \end{cases}$$

which is one of the classical models governing dynamics of nonlinear waves and describing interactions between high- and low- frequency waves, where n is the perturbed number density of the ion (in the low-frequency response), E is slow variation amplitude of the electric field intensity, c_s is the thermal traity transportation velocity of the electron-ion, and $\alpha \neq 0, \beta \neq 0, \delta \neq 0$ and c_s are constants.

Since $E(x,t)$ in (6) is a complex function we assume that

$$E(x,t) = U(\xi)\exp[i(kx - \omega t + \xi_0)],$$
$$n(x,t) = n(\xi), \qquad \xi = x - c_g t + \xi_1, \qquad (7)$$

where $U(\xi)$ is a real-valued function c_g, k and w are constants to be determined later, and ξ_0 and ξ_1 are constants. Substituting (7) into Eqs. (5), (6), we have

$$(c_g^2 - c_s^2)n'' = \beta(U^2)'' \qquad (8)$$
$$\alpha U'' + i(2\alpha k - c_g)U' + (\omega - \alpha k^2)U - \delta n U = 0. \qquad (9)$$

Integrating (8) twice with respect to ξ and taking integration constants to zero yields

$$n = \frac{\beta}{c_g^2 - c_s^2}U^2, \qquad c_g^2 - c_s^2 \neq 0. \qquad (10)$$

We assume that when $c_g^2 > c_s^2$(supersonic speed), n is positive; and when $c_g^2 < c_s^2$(subsonic speed), n is negative.

Substituting (10) into (9),

$$\alpha U'' + i(2\alpha k - c_g)U' + (\omega - \alpha k^2)U - \frac{\beta\delta}{c_g^2 - c_s^2}U^3 = 0. \qquad (11)$$

In view of (9) we assume that

$$c_g = 2\alpha k, \qquad \gamma = \omega - \alpha k^2.$$

Then Eq. (11) becomes the nonlinear ODE

$$U''(\xi) + k_3 U^3(\xi) + k_1 U(\xi) = 0, \qquad (12)$$

where

$$k_3 = \frac{\delta\beta}{\alpha(4\alpha^2k^2 - c_s^2)}, \qquad k_1 = \frac{\gamma}{\alpha}.$$

By balancing the highest order derivative term U'' with the nonlinear term U^3 in (12), we obtain $N = 1$ in (4). So we assume that Eq. (12) has solution in the form

$$U(\xi) = A_0 + A_1\left(\frac{F'}{F}\right), \qquad A_1 \neq 0. \qquad (13)$$

Using (13), we obtain

$$U^3 = A_0^3 + 3A_0^2A_1\left(\frac{F'}{F}\right) + 3A_0A_1^2\left(\frac{F'}{F}\right)^2 + A_1^3\left(\frac{F'}{F}\right)^3, \qquad (14)$$

$$U'' = A_1\left(\frac{F'''}{F} - 3\frac{F'F''}{F^2} + 2\left(\frac{F'}{F}\right)^3\right). \qquad (15)$$

Substituting (13) to (15) into Eq. (12) and setting the coefficients of F^j $(j = 0, -1, -2)$ to zero, we obtain

$$k_1A_0 + k_3A_0^3 = 0, \qquad (16)$$
$$A_1F''' + k_1A_1F' + 3k_3A_0^2A_1F' = 0, \qquad (17)$$
$$-3A_1F'F'' + 3k_3A_0A_1^2F'^2 = 0, \qquad (18)$$
$$2A_1F'^3 + k_3A_1^3F'^3 = 0. \qquad (19)$$

Eqs. (16) and (19) directly imply following solutions:

$$A_0 = \pm\sqrt{-\frac{k_1}{k_3}}, \quad A_1 = \pm\sqrt{-\frac{2}{k_3}}, \quad k_1 > 0, \ k_3 < 0.$$

Thus, Eqs. (17) and (18) become

$$F''' - 2k_1F' = 0, \qquad (20)$$
$$-F'' + \sqrt{2k_1}F' = 0. \qquad (21)$$

By substituting Eq. (21) into Eq. (20) we get

$$-\sqrt{2k_1}F'' + F''' = 0. \qquad (22)$$

The general solution of Eq. (22) is

$$F(\xi) = a_0 + a_1\xi + a_2\exp(\sqrt{2k_1}\xi),$$

where a_i ($i = 0,1,2$) are arbitrary constants. Thus, we have

$$U(\xi) = \pm\sqrt{-\frac{k_1}{k_3}} \pm \sqrt{-\frac{2}{k_3}}\left(\frac{a_1 + \sqrt{2k_1}a_2\exp(\sqrt{2k_1}\xi)}{a_0 + a_1\xi + a_2\exp(\sqrt{2k_1}\xi)}\right),$$

and

$$n(\xi) = \frac{\beta}{c_g^2 - c_s^2}\left(\pm\sqrt{-\frac{k_1}{k_3}} \pm \sqrt{-\frac{2}{k_3}}\left(\frac{a_1 + \sqrt{2k_1}a_2\exp(\sqrt{2k_1}\xi)}{a_0 + a_1\xi + a_2\exp(\sqrt{2k_1}\xi)}\right)\right)^2.$$

Now, the exact solution of Eqs.(5) and (6) have the form

$$E(x,t) = \pm\sqrt{-\frac{k_1}{k_3}} \pm \sqrt{-\frac{2}{k_3}}\left(\frac{a_1 + \sqrt{2k_1}a_2\exp\left(\sqrt{2k_1}(x - c_gt + \xi_1)\right)}{a_0 + a_1(x - c_gt + \xi_1) + a_2\exp\left(\sqrt{2k_1}(x - c_gt + \xi_1)\right)}\right) \times \exp(i(kx - \omega t + \xi_0)). \qquad (23)$$

and

$$n(x,t)=\frac{\beta}{c_g^2-c_s^2}$$

$$\times\left(\pm\sqrt{-\frac{k_1}{k_3}}\pm\sqrt{-\frac{2}{k_3}}\left(\frac{a_1+\sqrt{2k_1}a_2\exp\left(\sqrt{2k_1}(x-c_gt+\xi_1)\right)}{a_0+a_1(x-c_gt+\xi_1)+a_2\exp\left(\sqrt{2k_1}(x-c_gt+\xi_1)\right)}\right)\right)^2 \quad (24)$$

If $a_1=0$ and $a_0=a_2=1$, we have

$$E(x,t)=\pm\sqrt{-\frac{k_1}{k_3}}\left(2+\tanh\sqrt{\frac{k_1}{2}}(x-c_gt+\xi_1)\right)\times\exp(i(kx-\omega t+\xi_0))$$

and

$$n(x,t)=\frac{\beta}{c_g^2-c_s^2}\left[\sqrt{-\frac{k_1}{k_3}}\left(2+\tanh\sqrt{\frac{k_1}{2}}(x-c_gt+\xi_1)\right)\right]^2$$

Example. Solve the Zakharov equations by using the odified simplest equation method

$$\begin{cases} n_{tt}-n_{xx}=(|E|^2)_{xx},\\ iE_t+E_{xx}=nE,\end{cases}$$

Substituting $c_s^2=1$, $\beta=1$, $\alpha=1$ and $\delta=1$ in (23) and (24) gives

$$E(x,t)=\pm\sqrt{(\omega-k^2)(1-4k^2)}\pm\sqrt{2(1-4k^2)}$$
$$\times\left(\frac{a_1+\sqrt{2(\omega-k^2)}a_2\exp\left(\sqrt{2(\omega-k^2)}(x-c_gt+\xi_1)\right)}{a_0+a_1(x-c_gt+\xi_1)+a_2\exp\left(\sqrt{2(\omega-k^2)}(x-c_gt+\xi_1)\right)}\right)$$
$$\times\exp(i(kx-\omega t+\xi_0)).$$

and

$$n(x,t)=\frac{1}{4k^2-1}\left[\pm\sqrt{(\omega-k^2)(1-4k^2)}\pm\sqrt{2(1-4k^2)}\right.$$
$$\left.\times\left(\frac{a_1+\sqrt{2(\omega-k^2)}a_2\exp\left(\sqrt{2(\omega-k^2)}(x-c_gt+\xi_1)\right)}{a_0+a_1(x-c_gt+\xi_1)+a_2\exp\left(\sqrt{2(\omega-k^2)}(x-c_gt+\xi_1)\right)}\right)\right]^2.$$

3.2 The modified simplest equation method to the coupled Klein-Gordon-Zakharov equations

In this subsection, we consider the coupled nonlinear Klein-Gordon-Zakharov equations

$$\begin{cases} u_{tt}-c_0^2\nabla^2u+f_0^2u+\delta uv=0\\ v_{tt}-c_0^2\nabla^2v-\beta c_0^2\nabla^2|u|^2=0.\end{cases} \quad (25)$$

We seek its following wave packet solution

$$u(x,y,z,t)=U(\xi)\exp\left(i(kx+ly+nz-\Omega t)\right),$$

$$v(x,y,z,t)=V(\xi),\quad \xi=px+qy+rz-\omega t \quad (26)$$

where both $U(\xi)$ and $V(\xi)$ are real-valued functions k,l,n,Ω,p,q,r and ω are constants to be determined later. Substituting Eq. (26) into Eq. (25) yields

$$(w^2-c_0^2P^2)U''(\xi)+2i(\omega\Omega-c_0^2KP)U'(\xi)$$
$$-(w^2-c_0^2K^2-f_0^2)U(\xi)+\delta V(\xi)U(\xi)=0,$$
$$(w^2-c_0^2P^2)V''(\xi)-\beta P^2(U^2(\xi))''=0. \quad (27)$$

where

$$K=(k,l,n),\quad K^2=k^2+l^2+n^2,\quad P=(p,q,r),\quad P^2=p^2+q^2+r^2,\quad K.P=kp+lq+nr$$

In view of (27) we assume that

$$\omega\Omega=c_0^2K.P, \quad (28)$$

then (27) is reduced to

$$(w^2-c_0^2P^2)U''(\xi)-(w^2-c_0^2K^2-f_0^2)U(\xi)+\delta V(\xi)U(\xi)=0, \quad (29)$$

$$(w^2-c_0^2P^2)V''(\xi)-\beta P^2(U^2(\xi))''=0, \quad (30)$$

Integrating (30) once respect to ξ we get

$$(w^2-c_0^2P^2)V'(\xi)-\beta P^2(U^2(\xi))'=c_1, \quad (31)$$

where c_1 is integration constant. Because we find the special form of exact solutions for simplicity purpose, we take $c_1=0$ and integrating this formula once again, we have

$$V(\xi)=\frac{c_2}{(w^2-c_0^2P^2)}+\frac{\beta P^2}{(w^2-c_0^2P^2)}U^2(\xi), \quad (32)$$

where c_2 is integration constant. Substituting (32) into (29) yields

$$(w^2-c_0^2P^2)^2U''(\xi)+\left[(w^2-c_0^2P^2)\left((w^2-c_0^2K^2-f_0^2)\right)+\delta c_2\right]U(\xi)+\delta\beta P^2U^3(\xi)=0. \quad (33)$$

Eq. (33) can be expressed as

$$U''(\xi)+k_3U^3(\xi)+k_1U(\xi)=0 \quad (34)$$

where

$$k_1=\frac{\left[(w^2-c_0^2P^2)\left((w^2-c_0^2K^2-f_0^2)\right)+\delta c_2\right]}{(w^2-c_0^2P^2)^2},$$
$$k_3=\frac{\delta\beta P^2}{(w^2-c_0^2P^2)^2}$$

By balancing the highest order derivative term U'' with the nonlinear term U^3 in (34), we obtain $N=1$ in (4). So we assume that Eq. (34) has solution in the form

$$U(\xi)=A_0+A_1\left(\frac{F'}{F}\right),\quad A_1\neq 0. \quad (35)$$

Using (35), we obtain

$$U^3=A_0^3+3A_0^2A_1\left(\frac{F'}{F}\right)+3A_0A_1^2\left(\frac{F'}{F}\right)^2+A_1^3\left(\frac{F'}{F}\right)^3, \quad (36)$$

$$U''=A_1\left(\frac{F'''}{F}-3\frac{F'F''}{F^2}+2\left(\frac{F'}{F}\right)^3\right). \quad (37)$$

Substituting (35) to (37) into Eq. (34) and setting the coefficients of F^j $(j=0,-1,-2)$ to zero, we obtain

$$k_1A_0+k_3A_0^3=0, \quad (38)$$
$$A_1F'''+k_1A_1F'+3k_3A_0^2A_1F'=0, \quad (39)$$

$$-3A_1F'F'' + 3k_3A_0A_1^2F'^2 = 0, \qquad (40)$$
$$2A_1F'^3 + k_3A_1^3F'^3 = 0. \qquad (41)$$

Eqs. (38) and (41) directly imply following solutions:

$$A_0 = \pm\sqrt{-\frac{k_1}{k_3}}, \quad A_1 = \pm\sqrt{-\frac{2}{k_3}}, \quad k_1 > 0, \quad k_3 < 0.$$

Thus, Eqs. (39) and (40) become

$$F''' - 2k_1F' = 0, \qquad (42)$$

$$-F'' + \sqrt{2k_1}F' = 0. \qquad (43)$$

By substituting Eq. (43) into Eq. (42) we get

$$-\sqrt{2k_1}F'' + F''' = 0. \qquad (44)$$

The general solution of Eq. (44) is

$$F(\xi) = a_0 + a_1\xi + a_2\exp\left(\sqrt{2k_1}\xi\right)$$

where a_i ($i = 0,1,2$) are arbitrary constants.

Thus, we have

$$U(\xi) = \pm\sqrt{-\frac{k_1}{k_3}} \pm \sqrt{-\frac{2}{k_3}}\left(\frac{a_1 + \sqrt{2k_1}a_2\exp\left(\sqrt{2k_1}\xi\right)}{a_0 + a_1\xi + a_2\exp\left(\sqrt{2k_1}\xi\right)}\right),$$

and

$$V(\xi) = \frac{c_2}{(w^2 - c_0^2P^2)} + \frac{\beta P^2}{(w^2 - c_0^2P^2)}$$
$$\times\left(\pm\sqrt{-\frac{k_1}{k_3}} \pm \sqrt{-\frac{2}{k_3}}\left(\frac{a_1+\sqrt{2k_1}a_2\exp\left(\sqrt{2k_1}\xi\right)}{a_0+a_1\xi+a_2\exp\left(\sqrt{2k_1}\xi\right)}\right)\right)^2.$$

Now, the exact solution of Eq. (25) has the form

$$u(x,y,z,t) = \pm\sqrt{-\frac{k_1}{k_3}} \pm \sqrt{-\frac{2}{k_3}}$$
$$\times\left(\frac{a_1 + \sqrt{2k_1}a_2\exp\left(\sqrt{2k_1}(px + qy + rz - \omega t)\right)}{a_0 + a_1(px + qy + rz - \omega t) + a_2\exp\left(\sqrt{2k_1}(px + qy + rz - \omega t)\right)}\right)$$
$$\times\exp\left(i(kx + ly + nz - \Omega t)\right). \qquad (45)$$

and

$$v(x,y,z,t) = \frac{c_2}{(w^2 - c_0^2P^2)} + \frac{\beta P^2}{(w^2 - c_0^2P^2)}\left[\pm\sqrt{-\frac{k_1}{k_3}} \pm \sqrt{-\frac{2}{k_3}}\right.$$
$$\left.\times\left(\frac{a_1 + \sqrt{2k_1}a_2\exp\left(\sqrt{2k_1}(px + qy + rz - \omega t)\right)}{a_0 + a_1(px + qy + rz - \omega t) + a_2\exp\left(\sqrt{2k_1}(px + qy + rz - \omega t)\right)}\right)\right]^2 \qquad (46)$$

If $a_1 = 0$ and $a_0 = a_2 = 1$, we have

$$u(x,y,z,t) = \pm\sqrt{-\frac{k_1}{k_3}}\left(2 + \tanh\sqrt{\frac{k_1}{2}}(px + qy + rz - \omega t)\right)$$
$$\times\exp\left(i(kx + ly + nz - \Omega t)\right)$$

and

$$v(x,y,z,t) = \frac{c_2}{(w^2 - c_0^2P^2)} + \frac{\beta P^2}{(w^2 - c_0^2P^2)}$$
$$\times\left[\pm\sqrt{-\frac{k_1}{k_3}}\left(2 + \tanh\sqrt{\frac{k_1}{2}}(px + qy + rz - \omega t)\right)\right]^2$$

Example. Solve the Klein-Gordon-Zakharov equations by using the modified simplest equation method

$$\begin{cases} u_{tt} - u_{xx} + u + uv = 0, \\ v_{tt} - v_{xx} - (|u|^2)_{xx} = 0. \end{cases}$$

Substituting $c_0^2 = 1, f_0^2 = 1, \beta = 1, \delta = 1$ and $c_2 = 0$ in (45) and (46) gives

$$u(x,y,z,t) = \pm\sqrt{\frac{(\Omega^2 - k^2 - 1)}{(\omega^2 - p^2)}} \pm \sqrt{-\frac{2(\omega^2 - p^2)^2}{p^2}}$$
$$\times\left(\frac{a_1 + \sqrt{2\left(\frac{-\Omega^2 + k^2 + 1}{\omega^2 - p^2}\right)}a_2\exp\left(\sqrt{2\left(\frac{-\Omega^2 + k^2 + 1}{\omega^2 - p^2}\right)}(px + qy + rz - \omega t)\right)}{a_0 + a_1(px + qy + rz - \omega t) + a_2\exp\left(\sqrt{2\left(\frac{-\Omega^2 + k^2 + 1}{\omega^2 - p^2}\right)}(px + qy + rz - \omega t)\right)}\right)$$
$$\times\exp\left(i(kx + ly + nz - \Omega t)\right).$$

and

$$v(x,y,z,t) = \frac{P^2}{(w^2 - P^2)}\left[\pm\sqrt{\frac{(\Omega^2 - k^2 - 1)}{(\omega^2 - p^2)}} \pm \sqrt{-\frac{2(\omega^2 - p^2)^2}{p^2}}\right.$$
$$\left.\times\left(\frac{a_1 + \sqrt{2\left(\frac{-\Omega^2 + k^2 + 1}{\omega^2 - p^2}\right)}a_2\exp\left(\sqrt{2\left(\frac{-\Omega^2 + k^2 + 1}{\omega^2 - p^2}\right)}(px + q + rz - \omega t)\right)}{a_0 + a_1(px + qy + rz - \omega t) + a_2\exp\left(\sqrt{2\left(\frac{-\Omega^2 + k^2 + 1}{\omega^2 - p^2}\right)}(px + qy + rz - \omega t)\right)}\right)\right]^2$$

4. Conclusion

In this work, we obtained exact solutions of the Zakharov and the coupled Klein-Gordon-Zakharov equations by using the modified simplest equation method. The results show that this method is efficient.

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