Application of ICC Statistics and Their Hypothesis Tests

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***Abstract*:** The agreement of measuring methods is one of the main problems in medicine and others science in related to measure. In this article we deal with one of these methods and performance the statistical test and finally show their result. This method is interclass correlation coefficient.

***Keywords*:** Consistency, Absolute agreement, Interclass correlation coefficient (ICC).

1. Introduction

among other measuring methods, the ICC method is a perfect method for surviving the agreement. Because, by using ANOVA the total variance can separate to variance component and finally the main source is found. On the other hand, by using ICC we can find the variations which the difference among methods is its cause. At first we define the ICC, and then in each state we define the ICC according to data structures and find its unbiased estimators.

1.1 Interclass correlation coefficient

ICC is a kind of correlation coefficient, measuring the agreement and crassness of two or more samples. In fact, in ICC the agreement of referee(here, the referee can be the measuring machines) by analyzing that the cause of variations is the referees or other is obtained.

Now, the ICC is defined; the ICC is the part of variation which is not in related to referee. Therefore, the more ICC value, the less variation of referees is obtained then the agreement is increased. Suppose that there are *n* measurable topics, measured by *k* referees (table 1-4). This referees maybe measurable machines. The hypothesis is whether referees are agreed or disagree. In table (1-4) x_{ij} means the data from j^{th} referees on the i^{th} topic.

Table 1.

	Referee					
Topic	1	2	.	.	.	k
1	x_{11}	x_{12}	.	.	.	x_{1k}
2	x_{21}	x_{22}	.	.	.	x_{2k}
.	.	.				.
.	.	.				.
.	.	.				.
n	x_{n1}	x_{n2}	.	.	.	x_{nk}

2. Model fitting

model fitting is the first step for calculating the ICC;however it is on the base of ANOVA. For data of table 1-4 we have these states:

State1: the measurable topics are random samples from larger population. This is one way ANOVA.

State 2: the measurable topics and referees both are random samples and there is interaction between them. This is two way ANOVA.

State 2A: this is the same as state 2 which there is no interaction (two ways ANOVA).

State 3: In this state the random measurable topics and referees are registered, the model is mixture and there is interaction between them (two ways ANOVA).

State 3A: this state is the same as state 3 but there is no interaction (2 ways ANOVA) if the row variable be the only source of variances, the model is one way random effects.

This state occurs when the orders of gathering x_{ij} do not have any relation to *j*. the one way model is

$$x_{ij} = \mu + r_i + w_{ij} \begin{cases} i = 1,\dots,n \\ j = 1,\dots,k \end{cases}$$

If the column of the one systematic variation from source of variation is added, then the two ways model is better. These indicate that the referee factor is the random variable itself. Suppose the model

$$x_{ij} = \mu + r_i + c_j + (rc)_{ij} + e_{ij} \begin{cases} i = 1,\dots,n \\ j = 1,\dots,k \end{cases}$$

For two ways model. The table 2 presented the hypothesis for each state.

Table 2.

status	model	hypothesis
1: the model of one way random effect	$x_{ij} = \mu + r_i + w_{ij}$ $\begin{cases} i = 1,\dots,n \\ j = 1,\dots,k \end{cases}$	μ Is the population mean and fix. The row random effect is $r_i \overset{iid}{\sim} N(0,\sigma_r^2)$. The remain effect is $w_{ij} \overset{iid}{\sim} N(0,\sigma_w^2)$. The r_i and w_{ij} are mutually independent.
2: the model of two way random effects with interaction effect	$x_{ij} = \mu + r_i + c_j + (rc)_{ij} + e_{ij}$ $\begin{cases} i = 1,\dots,n \\ j = 1,\dots,k \end{cases}$	μ *and* r_i Are the same as state 1. The effect of column is $(c_j) \overset{iid}{\sim} N(0,\sigma_c^2)$. The row and column interaction is $(rc)_{ij} \overset{iid}{\sim} N(0,\sigma_{rc}^2)$. The effect or errors which $e_{ij} \overset{iid}{\sim} N(0,\sigma_\varepsilon^2)$. All effects are mutually independent.
2a: the model of two way random effects without interaction effect	$x_{ij} = \mu + r_i + c_j + e_{ij}$ $\begin{cases} i = 1,\dots,n \\ j = 1,\dots,k \end{cases}$	This is the same as state 2 which there is no interaction.
3: The two way conjugate model with interaction effect	$x_{ij} = \mu + r_i + c_j + (rc)_{ij} + e_{ij}$ $\begin{cases} i = 1,\dots,n \\ j = 1,\dots,k \end{cases}$	Is the same as state 2 whichthe c_j are fixed and $\sum c_j = 0$ and $\sum (rc)_{ij} = 0$ and the replacement parameter of σ_c^2 is $\theta_c^2 = \dfrac{\sum c_j^2}{k-1}$
3A: The two way conjugate model without interaction effect	$x_{ij} = \mu + r_i + c_j + e_{ij}$ $\begin{cases} i = 1,\dots,n \\ j = 1,\dots,k \end{cases}$	The same as state 3 which there is no interaction.

3. Variance analysis

In model one way random effect with considering the hypothesis of table 2, the variance of each observation is:

$$Var(x_{ij}) = \sigma_w^2 + \sigma_r^2$$

σ_w^2 and σ_r^2 are variance characters. Now, we find the expectation of them, MS_R and MS_W;

$$MS_w = \frac{1}{n(k-1)} SS_w = \frac{1}{n(k-1)} \sum_{i=1}^{n} \sum_{j=1}^{k} (x_{ij} - \bar{x}_{i.})^2$$

$$= \frac{1}{n(k-1)} \left[\sum_{i=1}^{n} \sum_{j=1}^{k} x_{ij}^2 - \frac{1}{k} \sum_{i=1}^{n} x_{i.}^2 \right] \quad (1)$$

$$= \frac{1}{n(k-1)} \left[\sum_{i=1}^{n} \sum_{j=1}^{k} x_{ij}^2 - 2k \sum_{i=1}^{n} \bar{x}_{i.}^2 + k \sum_{i=1}^{n} \bar{x}_{i.}^2 \right]$$

Therefore, we have:

$$E(MS_w) = \frac{1}{n(k-1)} \left[E\left(\sum_{I=1}^{n} \sum_{J=1}^{k} x_{ij}^2 \right) - \frac{1}{k} E\left(\sum_{i=1}^{n} x_{i.}^2 \right) \right] \quad (2)$$

However because $E(r_i) = E(w_{ij}) = 0$, $E(w_{ij}^2) = \sigma_w^2$, $E(r_i^2) = \sigma_r^2$, we have:

$$E\left(\sum_{i=1}^{n} \sum_{j=1}^{k} x_{ij}^2 \right) = E\left[\sum_{i=1}^{n} \sum_{j=1}^{k} (\mu + r_i + w_{ij})^2 \right]$$

$$=k\sum_{i=1}^{n}E(\mu+r_i)^2+\sum_{i=1}^{n}\sum_{j=1}^{k}E\left(w_{ij}^2\right)$$

$$=E\left[k\sum_{i=1}^{n}(\mu+r_i)^2+2\sum_{i=1}^{n}(\mu+r_i)w_{i.}+\sum_{i=1}^{n}\sum_{j=1}^{k}w_{ij}^2\right]$$

$$=nk\left(\mu^2+\sigma_r^2\right)+nk\sigma_w^2 \qquad (3)$$

And also

$$\frac{1}{k}E\left(\sum_{i=1}^{n}x_{i.}^2\right)=\frac{1}{k}E\left[\sum_{i=1}^{n}\left(\sum_{j=1}^{k}(\mu+r_i+w_{ij})\right)^2\right]$$

$$=\frac{1}{k}E\left[\sum_{i=1}^{n}(k\mu+kr_i+w_{i.})^2\right]=\frac{1}{k}\sum_{i=1}^{n}E(k\mu+kr_i+w_{i.})^2$$

$$=\frac{1}{k}\sum_{i=1}^{n}\left(k^2\mu^2+k^2\sigma_r^2+k\sigma_w^2\right)$$

$$=nk\left(\mu^2+\sigma_r^2\right)+n\sigma_w^2 \qquad (4)$$

Now, by replacing (3), (4) in relation (2), we have:

$$E(MS_w)=\frac{1}{n(k-1)}\left[nk(\mu^2+\sigma_r^2)+nk\sigma_w^2-nk(\mu^2+\sigma_r^2)-n\sigma_w^2\right]=\sigma_w^2 \qquad (5)$$

Therefore MS_W is an unbiased estimator for σ_w^2. Now, we have:

$$MS_R=\frac{k\sum_{i=1}^{n}(\bar{x}_{i.}-\bar{x}_{..})^2}{n-1}=\frac{k\sum_{i=1}^{n}\bar{x}_{i.}^2-2x_{..}\bar{x}_{..}+nk\,\bar{x}_{..}^2}{n-1}$$

$$=\frac{1}{n-1}\left[\sum_{i=1}^{n}\frac{x_{i.}^2}{k}-\frac{x_{..}^2}{nk}\right]$$

Then we have:

$$E(MS_R)=\frac{1}{n-1}\left[E\left(\sum_{i=1}^{n}\frac{x_{i.}^2}{k}\right)-E\left(\frac{x_{..}^2}{nk}\right)\right] \qquad (6)$$

On the other hand

$$E\left(\frac{x_{..}^2}{nk}\right)=\frac{1}{nk}E\left[\sum_{i=1}^{n}\sum_{j=1}^{k}(\mu+r_i+w_{ij})\right]^2$$

$$=\frac{1}{nk}E\left(nk\mu+k\sum_{i=1}^{n}r_i+w_{..}\right)^2=nk\mu^2+k\sigma_r^2+\sigma_w^2 \quad (7)$$

By replacing (4), (7) in relation (6) we have:

$$E(MS_R)=\sigma_w^2+k\sigma_r^2 \qquad (8)$$

Similarly, by considering the hypothesis of models the expectation and variance is received. The other results are drawn in table (3) [2].

Table 3. mean square for models variance analysis

State	The source of variation	df	MS	EMS
1	Among rows	$(n-1)$	MS_R	$k\sigma_r^2+\sigma_w^2$
	Inside rows	$n(k-1)$	MS_W	σ_w^2
2	Among rows	$(n-1)$	MS_R	$k\sigma_r^2+\sigma_{rc}^2+\sigma_e^2$
	Inside rows	$n(k-1)$	MS_W	$\sigma_c^2+\sigma_{rc}^2+\sigma_e^2$
	Among column	$(k-1)$	MS_C	$n\sigma_c^2+\sigma_{rc}^2+\sigma_e^2$
	Error	$(n-1)(k-1)$	MS_E	$\sigma_{rc}^2+\sigma_e^2$
2A	Among rows	$(n-1)$	MS_R	$k\sigma_r^2+\sigma_e^2$
	Inside rows	$n(k-1)$	MS_W	$\sigma_c^2+\sigma_e^2$
	Among column	$(k-1)$	MS_C	$n\sigma_c^2+\sigma_e^2$
	Error	$(n-1)(k-1)$	MS_E	σ_e^2
3	Among rows	$(n-1)$	MS_R	$k\sigma_r^2+\sigma_e^2$
	Inside rows	$n(k-1)$	MS_W	$\theta_c^2+\frac{k}{k-1}\sigma_{rc}^2+\sigma_e^2$
	Among column	$(k-1)$	MS_C	$n\theta_c^2+\frac{k}{k-1}\sigma_{rc}^2+\sigma_e^2$
	Error	$(n-1)(k-1)$	MS_E	$\frac{k}{k-1}\sigma_{rc}^2+\sigma_e^2$

3A	Among rows	$(n-1)$	MS_R	$k\sigma_r^2+\sigma_e^2$
	Inside rows	$n(k-1)$	MS_W	$\theta_c^2+\sigma_e^2$
	Among column	$(k-1)$	MS_C	$n\theta_c^2+\sigma_e^2$
	Error	$(n-1)(k-1)$	MS_E	σ_e^2

4. Finding an unbiased estimator for (ICC) in one way and two way models

For one way model the ICC coefficient is the ratio of row variances to total variances.

$$\rho=\frac{\sigma_r^2}{\sigma_r^2+\sigma_w^2} \tag{9}$$

By considering table 3 the MS_w is an unbiased estimator for σ_w^2. Now we find an unbiased estimator for σ_r^2 :

$$\frac{E(MS_R-MS_w)}{k}=\sigma_r^2 \tag{10}$$

Therefore $\frac{MS_R-MS_w}{k}$ is an unbiased estimator for σ_r^2 .

Now, if in relation (10) replaced all unbiased estimators instead of their variances then and unbiased estimator for ρ will be found. Its estimator indicated by ICE(1)

$$ICC(1)=\frac{\frac{1}{k}(MS_R-MS_w)}{\frac{1}{k}(MS_R-MS_w)+MS_w}=\frac{MS_R-MS_w}{MS_R-(k-1)MS_w} \tag{11}$$

For two way models in state 2, the ICC coefficient for consistency is the ratio of variances:

$$\rho=\frac{\sigma_r^2}{\sigma_r^2+\sigma_{rc}^2+\sigma_e^2} \tag{12}$$

By considering table (3) we can find each of unbiased estimators for variances. Therefore, an unbiased estimator for (12) is:

$$ICC(c,1)=\frac{MS_R-MS_E}{MS_R+(k-1)MS_E} \tag{13}$$

In this way, the ICC and their unbiased estimators could be fined for one way and two way models. Calculating and inferring of ICC is shown in table (4). In this table there is two kinds of ICC. Both of them measure the correlation, however the first one by consistency and the second one by absolute agreement. In other word, their difference is in their formula which is obvious in their denominators.

In definition of ICC for consistency, we suppose that the column variances variation source does not have any relation to total variance. Therefore, this factor is ignored in denominator. Because this kind of ICC measures the consistency of data, it is more similar to Pierson correlation coefficient(r).

Another kind of ICC (absolute agreement) measures the real agreement of data. For instance, as data we have (2, 4), (4, 6), (6, 8) therefore[ICC(c,1)=1] , [ICC(A,1)=0.67].

Since, on the base of absolute agreement the ICC value is low because, the interval of data is considering. In table (14) the C and A letters indicate the agreement from consistency and absolute agreement respectively. In one way models there is no existence kind (C) and in two way models they are shown by (A, 1), (C, 1).

Table 4. ICC for one way and two way models

Model	**The formula**	**sign**	**interpretation**
The one way model 1	$\frac{MS_R-MS_w}{MS_R+(k-1)MS_w}$	$ICC(1)$	Measuring the degree of absolute agreement among data.
The two way random model $\begin{cases}2\\2A\end{cases}$	$\frac{MS_R-MS_E}{MS_R+(k-1)MS_E}$	$ICC(c,1)$	Measuring the degree of consistency among data, both row and column are random.
The two way random model $\begin{cases}2\\2A\end{cases}$	$\frac{MS_R-MS_E}{MS_R+(k-1)MS_E+\frac{k}{n}(MS_C-MS_E)}$	$ICC(A,1)$	Measuring the degree of absolute agreement among data, both row and column are random.
The conjugatetwo way random model	$\frac{MS_R-MS_E}{MS_R+(k-1)MS_E}$	$ICC(c,1)$	Degree of consistency among data, the column factor is fixed.

$\begin{cases} 3 \\ 3A \end{cases}$			
The conjugate two way random model $\begin{cases} 3 \\ 3A \end{cases}$	$\dfrac{MS_R - MS_E}{MS_R + (k-1)MS_E + \frac{k}{n}(MS_C - MS_E)}$	$ICC(A,1)$	Measuring the degree of absolute agreement among data.

5. Statistical test for types of ICC

Generally, we interested in testing $H_0 : \rho = \rho_0$ for different kinds of ICC. Now, for each kind (one way, two ways) we find the fair statistics test as bellow

5.1 one way random effect model (state1)

In this case the statistical hypothesis is

$H_0 : ICC(1) = \dfrac{\sigma_r^2}{\sigma_r^2 + \sigma_w^2} = \rho_0$. First consider $\dfrac{MS_R}{MS_w}$ for making statistical test. It is obviously that if the square means are divided to their expectation then the distribution will be χ^2. Therefore, for the distribution of $\dfrac{MS_R}{MS_w}$ will be F.

By considering table (3) we have $\{E(MS_w) = \sigma_w^2\}$ and $\{E(MS_R) = k\sigma_r^2 + \sigma_w^2\}$ then:

$$F = \left[\frac{MS_R}{k\sigma_r^2 + \sigma_w^2} \Big/ \frac{MS_w}{\sigma_w^2} \right] = \frac{MS_R}{MS_w} \cdot \frac{\sigma_w^2}{k\sigma_r^2 + \sigma_w^2} \quad (14)$$

On the other hand, under H_0, we have:

$$\frac{k\sigma_r^2 + \sigma_w^2}{\sigma_w^2} = \frac{1 + (k-1)\rho_0}{1 - \rho_0} \quad (15)$$

Therefore under H_0 the statistical test is:

$$F = \frac{MS_R}{MS_w} \cdot \frac{1 - \rho_0}{1 + (k-1)\rho_0} \quad (16)$$

With $F_{(n-1)}$ distribution for nominator and $F_{n(k-1)}$ distribution for denominator.

5.2 two ways random effect models with interaction effect (state 2)

A: the statistical test for consistency:

The statistical hypothesis is $H_0 : ICC(c,1) = \dfrac{\sigma_r^2}{\sigma_r^2 + \sigma_{rc}^2 + \sigma_e^2} = \rho_0$. The $\left[\dfrac{MS_R}{E(MS_R)} \Big/ \dfrac{MS_E}{E(MS_E)} \right]$ has a Fisher distribution and it is considered as statistical test. By considering table (3) we have $\{E(MS_R) = k\sigma_r^2 + \sigma_{rc}^2 + \sigma_e^2\}$ and $\{E(MS_w) = \sigma_{rc}^2 + \sigma_e^2\}$, therefore we have:

$$F = \frac{MS_R}{MS_E} \cdot \frac{\sigma_{rc}^2 + \sigma_e^2}{k\sigma_r^2 + \sigma_{rc}^2 + \sigma_e^2}$$

On the other hand, under H_0 we have:

$$1 - \rho = \frac{\sigma_{rc}^2 + \sigma_e^2}{\sigma_r^2 + \sigma_{rc}^2 + \sigma_e^2} \Rightarrow$$

$$k\sigma_r^2 + \sigma_{rc}^2 + \sigma_e^2 = \frac{[k\rho_0 + (1 - \rho_0)](\sigma_{rc}^2 + \sigma_e^2)}{1 - \rho_0}$$

$$\Rightarrow \frac{k\sigma_r^2 + \sigma_{rc}^2 + \sigma_e^2}{\sigma_{rc}^2 + \sigma_e^2} = \frac{1 + (k-1)\rho_0}{1 - \rho_0} \quad (18)$$

Then under H_0, the fair statistical test is

$F = \dfrac{MS_R}{MS_w} \cdot \dfrac{1 - \rho_0}{1 + (k-1)\rho_0}$ with $F_{(n-1)}$ distribution for denominator and $F_{(n-1)(k-1)}$ distribution for nominator.

B: Statistical test for absolute agreement:

The statistical hypothesis is

$$H_0 : ICC(A,1) = \frac{\sigma_r^2}{\sigma_r^2 + \sigma_c^2 + \sigma_{rc}^2 + \sigma_e^2} = \rho_0 .$$

Considering $\dfrac{MS_R}{aMS_C + bMS_E}$ for statistical test. If dividing the nominator and denominator intoexpectation then we will have the F distribution. By considering table (3) we have $E(MS_R) = k\sigma_r^2 + \sigma_{rc}^2 + \sigma_e^2$, $E(MS_E) = \sigma_{rc}^2 + \sigma_e^2$ and $E(MS_C) = n\sigma_c^2 + \sigma_{rc}^2 + \sigma_e^2$. Then:

$$F = \frac{MS_R}{aMS_c + bMS_E} \cdot \frac{a(n\sigma_c^2 + \sigma_{rc}^2 + \sigma_e^2) + b(\sigma_{rc}^2 + \sigma_e^2)}{(k\sigma_r^2 + \sigma_{rc}^2 + \sigma_e^2)} \quad (19)$$

On the other hand, under H_0:

$$k\sigma_r^2 + \sigma_{rc}^2 + \sigma_e^2 = \left[1 + \frac{k\rho_0}{1 - \rho_0} - \frac{k\rho_0}{n(1 - \rho_0)} \right](\sigma_{rc}^2 + \sigma_e^2) + \left[\frac{k\rho_0}{n(1 - \rho_0)} \right](n\sigma_c^2 + \sigma_{rc}^2 + \sigma_e^2)$$

Therefor by fair choosing of $a=\frac{k\rho_0}{n(1-\rho_0)}$ and $b=\left[1+\frac{k\rho_0}{1-\rho_0}-\frac{k\rho_0}{n(1-\rho_0)}\right]$, the statistical test is

$$F=\frac{MS_R}{aMS_c+bMS_E} \qquad (21)$$

With F $_{(n-1)}$ distribution for nominator but for finding the freedom degree of denominator use the Satterthwaitetheorem as bellow:

Theorem 1: Suppose $MS_1, MS_2, MS_3, \ldots$ are the square means with freedom of $r_1, r_2, r_3, \ldots$, with real numbers $a_1, a_2, a_3, \ldots$, respectively. Therefore, the linear combination $a_1MS_1+a_2MS_2+\ldots$ has the χ^2 distribution with freedom degree as bellow [4]:

$$r=\frac{(a_1MS_1+a_2MS_2+\ldots)^2}{\frac{(a_1MS_1)^2}{r_1}+\frac{(a_2MS_2)^2}{r_2}+\ldots} \qquad (22)$$

Therefore, with considering the linear combination

$$\nu=\frac{(aMS_C+bMS_E)^2}{\frac{(aMS_C)^2}{k-1}+\frac{(bMS_E)^2}{(n-1)(k-1)}} \qquad (23)$$

By continuing the above method, the statistical test for other stats is found:

Table 5. the statistical test for all kind of ICC

State	The statistical tests	Freedom degree
State 1	$F=\frac{MS_R}{MS_w}\cdot\frac{1-\rho_0}{1+(k-1)\rho_0}$	$[(n-1), n(k-1)]$
State 2,2A,3,3A for consistency (kind c)	$F=\frac{MS_R}{MS_E}\cdot\frac{1-\rho_0}{1+(k-1)\rho_0}$	$[(n-1),(n-1)(k-1)]$
State 2,2A,3,3A for absolute agreement (kind a)	$F=\frac{MS_R}{aMS_c+bMS_E}$ $a=\frac{k\rho_0}{n(1-\rho_0)}$ $b=\left[1+\frac{k\rho_0}{1-\rho_0}-\frac{k\rho_0}{n(1-\rho_0)}\right]$	$[(n-1),\nu]$ $\nu=\frac{(aMS_C+bMS_E)^2}{\frac{(aMS_C)^2}{k-1}+\frac{(bMS_E)^2}{(n-1)(k-1)}}$

6. Numerical example

The data in table (6) show the blood pressure of n=27 person who are measured by k=6 tools (A, B, C, D, E, F). Each factor are random and without interaction.

Table 6. the data of blood pressure

sub	A	B	C	D	E	F	$x_{i.}$
1	*100*	*122*	*208*	*190*	*166*	*167*	*953*
2	*108*	*121*	*94*	*103*	*146*	*173*	*745*
3	*76*	*95*	*114*	*131*	*204*	*228*	*848*
4	*108*	*127*	*126*	*131*	*96*	*77*	*665*
5	*124*	*140*	*124*	*126*	*134*	*154*	*802*
6	*122*	*139*	*110*	*121*	*138*	*154*	*784*
7	*116*	*122*	*90*	*97*	*134*	*145*	*704*
8	*114*	*130*	*106*	*116*	*156*	*200*	*822*
9	*100*	*119*	*218*	*215*	*124*	*188*	*964*
10	*108*	*126*	*130*	*141*	*114*	*149*	*768*
11	*100*	*107*	*136*	*153*	*112*	*136*	*744*
12	*108*	*123*	*100*	*113*	*112*	*128*	*684*
13	*112*	*131*	*100*	*109*	*202*	*204*	*858*
14	*104*	*123*	*124*	*145*	*132*	*184*	*812*
15	*106*	*127*	*164*	*192*	*158*	*163*	*910*
16	*122*	*142*	*100*	*112*	*88*	*93*	*657*
17	*100*	*104*	*136*	*152*	*170*	*178*	*840*
18	*118*	*117*	*114*	*141*	*182*	*202*	*874*
19	*140*	*139*	*148*	*206*	*112*	*162*	*907*
20	*150*	*143*	*160*	*151*	*120*	*227*	*951*
21	*166*	*181*	*84*	*112*	*110*	*133*	*786*
22	*148*	*149*	*156*	*162*	*112*	*202*	*929*
23	*174*	*173*	*110*	*117*	*154*	*158*	*886*
24	*174*	*160*	*100*	*119*	*116*	*124*	*793*

25	140	158	100	136	108	114	756
26	128	139	86	112	106	137	708
27	146	153	106	120	122	121	768
$x_{.j}$	3312	3610	3344	3723	3628	4301	21918

While both factors are random (state 2A) , first confirm the two way ANOVA for calculating the agreement on the base of ICC.

$$SS_R = \tfrac{1}{k}\sum_{i=1}^{n} x_{i.}^2 - \frac{x_{..}^2}{nk} = 33129.778 \quad ,$$

$$SS_c = \tfrac{1}{n}\sum_{j=1}^{k} x_{.j}^2 - \frac{x_{..}^2}{nk} = 23668.15$$

$$SS_{total} = \sum_{i=1}^{n}\sum_{j=1}^{k} x_{ij}^2 - \frac{x_{..}^2}{nk} = 167747.778 \quad ,$$

$$SS_w = SS_{total} - SS_R = 132618$$

$$SS_E = SS_w - SS_c = 108949.85$$

Then the ANOVA table is

Table 7. Analysis Variance

The source of variation	SS	df	MS
Among students	35129.778	26	1351.145308
Inside students	132618	135	982.355555
Among referees	23668.15	5	4733.63
Error	108949.85	130	838.0757692

Therefore we have:

$$ICC(c,1) = \frac{MS_R - MS_E}{MS_R + (k-1)MS_E} = 0.092586358$$

$$ICC(A,1) = \frac{MS_R - MS_E}{MS_R + (k-1)MS_E + \frac{k}{n}(MS_C - MS_E)}$$

$$= 0.080076993$$

For instance, suppose that $H_0 : \frac{\sigma_r^2}{\sigma_r^2 + \sigma_c^2 + \sigma_e^2} = 0$

On the base of table (5) we have:

$$a = \frac{k\rho_0}{n(1-\rho_0)} = 0 \, ,$$

$$b = \left[1 + \frac{k\rho_0}{1-\rho_0} - \frac{k\rho_0}{n(1-\rho_0)}\right] = [1+0] = 1$$

$$F = \frac{MS_R}{aMS_c + bMS_E} = \frac{1351.145308}{838.0757692} = 1.612199467$$

The value of this statistical test is less than the value of table $(F_{0.975(26,130)} = 1.748)$, and then the H_0 hypothesis is not rejected at the level of 0.05.

References

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