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Control Intervention in Meningitis Transmission on Children in North-Western Nigeria

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***Abstract*:** In this paper, mathematical model for the transmission dynamics of meningitis incorporated with best control strategies of educational campaign and vaccination is formulated. The model describes meningitis transmission designed into six compartment which leads to a linear system of differential equations, the model used data on meningitis outbreaks on children in the northwestern Nigeria. The model distinguished between female and male children population: between infected females and males, recovered with deficiency and without deficiency. Equilibrium points are found and their stability based on a threshold value R_0 is investigated as 9.4 which is greater than 1, therefore, since R_0 is greater than 1 the endemic equilibrium is stable both locally and globally. The numerical simulations and graphical solutions of the differential equations were carried out with Matlab application software.

***Keywords*:** meningitis, educational campaign,vaccination and threshold value.

1. Introduction

Minimizing the transmission of infectious diseases is a core function of public health to find out how serious the disease is, the transmission as well as to obtain optimal solution for the disease transmission. Mathematical modelling is useful in modelling disease transmission and sighting control measures of epidemic to end or reduce the rate of infection as well as factors stimulating a particular disease [1]. The optimal control model for epidemic diseases were formulated by many authors, depending on the type of diseases, emphasize on disease spreads, prediction and control [2]. Basically, one of the fundamentals of mathematical modelling of epidemics is to obtain optimal control for the disease spreads [3]. Another study carried by Isere et al. [4] on optimal control model for the outbreak of disease infection in Nigeria, two models were used: first model was to describe the bacterial population and its interaction with the population susceptible while the second was to control by means of optimal control.

Asmoa et al., [5] conducted modelling on Rabies transmission dynamics in order to find the optimal control of the disease spreads. According to [6] a susceptible, Exposed, infected and recovered (SEIR) type of control model described the dynamics of Ebola epidemics in a population of constant size, applied the theory of pontryagin's maximum principles and incorporated the control parameters. Therefore, the researcher employed the used of conjugate vaccines as a control against the most common pathogens which has been crucial in preventing and control of meningitis infection in children [7].

2. Materials and Methods

A mathematical model which will be formulated using differential equations based on the epidemiological compartment modeling used is the proposed S, C, I_F, I_M, R_D, R_N model for this paper, with educational campaign and vaccination as a control strategy, which will be incorporated into the model. The computer software package that will be used to solve the differential equation model numerically is Matrix Laboratory (Matlab R2014a). Data collected for the 2017 meningitis outbreaks in the north western Nigeria will be used and numerical simulations of the model will be conducted using Matlab application software.

3. Model Formulation and Analysis

The focus of this study is to investigate an effective strategy to control the spread of meningitis infection in the northwestern Nigeria in the S; C; I_F; I_M; R_D; R_N model. Two controls representing the effort that reduces the contact carrier rate and infectious classes between male and female children, educational campaign and vaccination are considered in order to minimize the number of carrier and infected individuals during the course of an epidemic.

This paper, consider the S; C; I_F; I_M; R_D; R_N epidemiological model with the assumption that educational campaign and vaccination incorporated are the only control strategies in addition, natural death and death rates due to infection being unequal. The deterministic, compartmental mathematical model is formulated to describe the transmission dynamics of meningitis infection in the northwestern Nigeria. It is also assumed that, the population is heterogeneous. That is no individuals that make up of population can be grouped into different compartment or groups according to their epidemiological class and is to be taken is constant, the population size in a compartment is differentiable with respect to time and deterministic.

In order words that the changes in population of a compartment can be calculated using only using history to developed the model. Natural deaths in each compartment and death due to meningitis only will be considered not of any other cause. The proportions of the population of children

are immunized against meningitis infection through vaccination. And the population mixed homogeneously. That is all susceptible individuals are equally likely to be infected by infectious individuals in case of contact.

3.1 Description of Variables and Parameters

The following tables describe the variables and parameters used in this model:

Table 1. Variables used in the model

Variables	Description	Initial Value	Source
$S(t)$	Susceptible Population	15, 885,000	NCDC, 2017
$I_F(t)$	Infected females	4,080	NCDC, 2017
$I_M(t)$	Infected Males	4,732	NCDC, 2017
$C(t)$	Natural Carrier	8,825,000	NCDC, 2017
$R_N(t)$	Recovered children with-out disability	5,480	NCDC, 2017
$R_D(t)$	Recovered children with disability	2,451	NCDC, 2017

Table 2. Parameters used in the model

Model Parameters	Description	Value	Source
λ	Recruitment rate	0.41	Estimated
α	Death rate due to Meningitis	0.01	NCDC, 2017
μ	Natural death rate	0.02	NCDC, 2017
$\eta 1$	rate of carrier contact	0.4	Estimated
$\eta 2$	rate of female contact	0.3	Estimated
$\eta 3$	rate of male contact	0.3	Estimated
$\psi 1$	rate of return to susceptible class from R_D	0.045	Estimated
$\psi 2$	rate of return to susceptible class from R_N	0.055	Estimated
δ	rate of moving from R_D to R_N	0.05	Estimated
$\rho 1$	rate of female recovery from infectious class	0.045	Estimated
$\rho 2$	rate of male recovery from infectious class	0.035	Estimated
κ	rate of moving to carrier naturally	0.25	Coen, 2000
ξ	educational campaign	0.10	Assume
ζ	vaccination	0.20	Assume
ϖ	rate of return from carrier to susceptible	0.03	Estimated

4. Compartmental Diagram

In this research, the population is divided into six disease-state compartments such as susceptible, carrier, infected female, infected male, recovered with deficiency and recovered without deficiency represented by S; C; I_F; I_M; R_D; R_N respectively. In which the model considered was S; C; I_F; I_M; R_D; R_N model: susceptible individuals (S), people who can catch the disease; carrier individuals (C), people whose body is a host for the infectious agent and are yet able to transmit the disease; infectious (infective) individuals I_F, and I_M, people who have the disease and can transmit the disease; recovered individuals R_D, and R_N, proportion of people who have recovered from the disease with disability and without disability. It is however, assume that an individual can be infected only through contacts with infectious individuals and that immunity is permanent.

Thus, the compartmental diagram for the deterministic model is as follows;

The model equations are given below:

5. Model Equation

The transitions between model classes can now be expressed by the following system of first order differential equations:

$$\begin{aligned}
\frac{dS}{dt} =& \lambda - \xi - \zeta + \varpi C + \eta_1 C + \eta_2 I_F + \eta_3 I_M - \beta S - \epsilon_1 S \\
& - \epsilon 2S + \psi_1 R_D + \psi_2 R_N - \mu S \\
\frac{dC}{dt} =& \beta S - \eta_1 C + \xi - \kappa_1 C - \kappa_2 C - \mu C \\
\frac{dI_f}{dt} =& \kappa_1 C + \epsilon_1 S + \zeta - \eta_2 I_F - \rho_2 I_F - \rho_1 I_F - \alpha I_F - \mu I_F \\
\frac{dI_m}{dt} =& \kappa_2 C + \epsilon_2 S + \zeta(1-\zeta) - \eta_3 I_M - \rho_2 I_M - \rho_1 I_M \\
& - \alpha I_M - \mu I_M \\
\frac{dR_d}{dt} =& \rho_2 I_F + \rho_2 I_M - \psi_1 R_D - \delta R_D - \mu R_D \\
\frac{dR_n}{dt} =& \rho_1 I_F + \rho_1 I_M + \delta R_D - \psi_2 R_N - \mu R_N
\end{aligned} \tag{1}$$

6. Basic Properties of the Model

6.1 Feasible region

Since the model monitors human population, all associated parameters of the model and state variables are assumed to be non-negative $t = 0$. It is quite simple to show that the state variables of the model remain non-negative for all non-negative initial conditions.

Lemma 1: The closed Ω is positively invariant and attracting.

Proof: Adding Eq. (1) gives the rate of change of the total population can be writen as:

$$N(t) = S(t) + C(t) + IF(t) + IM(t) + RD(t) + RN(t)$$

therefore:

$$\frac{\partial N}{\partial t} = \lambda - \alpha N \qquad \square$$

Thus, the total number of human population (N) is bounded by $\frac{\lambda}{\alpha}$, so that $\frac{\partial N}{\partial t} = 0$ whenever $N(t) = \frac{\lambda}{\alpha}$. It can be shown that $N(t) = \frac{\lambda}{\alpha} + (N_0 - \frac{\lambda}{\alpha})e^{\alpha t}$. In particular $N(t) = \frac{\lambda}{\alpha}$, if $N(0) = \frac{\lambda}{\alpha}$.

Hence, the region Ω is positively invariant and attract all solutions in R^6_+

7. Existence and uniqueness of solution

For the mathematical model to predict the future of the system from its current state at time the initial value problem (IVP)

$$x' = f(t, x), x(t_0) = x_0. \tag{2}$$

Must have a solution that exist and also unique. In this section, the conditions for the existence and uniqueness of solution for the model system of equation shall be established.

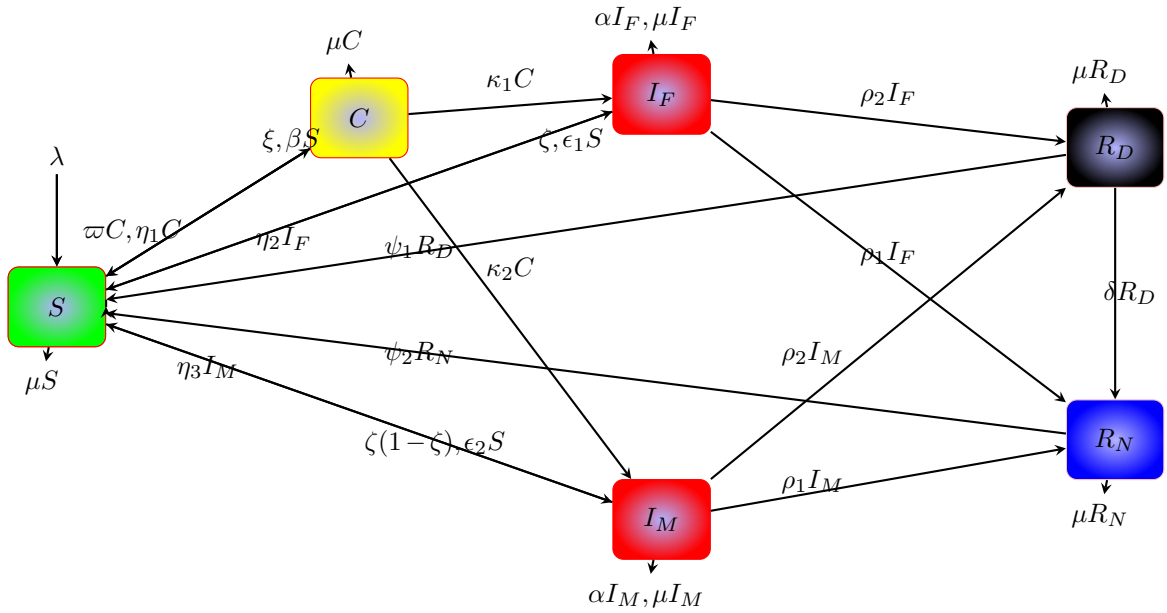

Figure 1. The compartmental diagram describing bacterial meningitis transmission dynamics within the population. The six squares represent the six compartments of individuals, such as, susceptible, carrier, infected male, infected female, recovered with deficiency and recovered without deficiency. The movement between the compartments is indicated by the continuous arrows, with the consideration that carrier measures lies in, $0 < C; C \leq 1$.

Let

$$f_{1x}(t,x) = \lambda - \xi - \zeta + \varpi C + \eta_1 C + \eta_2 I_F + \eta_3 I_M - \beta S - \epsilon_1 S - \epsilon 2S + \psi_1 R_D + \psi_2 R_N - \mu S \quad (3)$$

$$f_{2x}(t,x) = \beta S - \eta_1 C + \xi - \kappa_1 C - \kappa_2 C - \mu C \quad (4)$$

$$f_{3x}(t,x) = \kappa_1 C + \epsilon_1 S + \zeta - \eta_2 I_F - \rho_2 I_F - \rho_1 I_F - \alpha I_F - \mu I_F \quad (5)$$

$$f_{4x}(t,x) = \kappa_2 C + \epsilon_2 S + \zeta(1-\zeta) - \eta_3 I_M - \rho_2 I_M - \rho_1 I_M - \alpha I_M - \mu I_M \quad (6)$$

$$f_{5x}(t,x) = \rho_2 I_F + \rho_2 I_M - \psi_1 R_D - \delta R_D - \mu R_D \quad (7)$$

$$f_{6x}(t,x) = \rho_1 I_F + \rho_1 I_M + \delta R_D - \psi_2 R_N - \mu R_N \quad (8)$$

so that

$$x' = f(x,t), x(x_0) = f(x) \quad (9)$$

Theorem 1 (Momoh et al, 2013): Let D' denotes the region

$$|t - t_0| \leq a \|x - x_0\| \leq b \qquad x = (x_1, x_2, \ldots, x_n)$$

$$x_0 = (x_{10}, x_{20}, \ldots, x_{no}) \quad (10)$$

and suppose that $f(t,x)$ satisfies the Lipchitz condition.

$$\|f(t,x_1) - f(t,x_2)\| \leq K \|x_1 - x_2\| \quad (11)$$

whenever the pairs (t,x) and (x_1, x_2) belongs to D' where K is positive constant. Then there exist the constant $\delta > 0$ such that there exist a unique continuous vector solution $\bar{x}_t$ of the system in the interval $|t - t_0| \leq \delta$. It is satisfied by the requirement that $\frac{\partial f_i}{\partial x_j}, i,j = 1,2,\ldots,n$ be continuous and bounded in D'.

Lemma 2: If $f(t,x)$ has continuous partial derivatives $\frac{\partial f_i}{\partial x_j}$ on a bounded closed covex domain R, then it satisfies a Lipchitz condition in R. Being interested in the region

$$1 \leq \xi \leq R \quad (12)$$

then, looking for a bounded solution of the form

$$0 \leq R \leq \infty \quad (13)$$

and shall prove the following existence theorem.

Theorem 2: Let D' denote the region defined in such that above hold. Then there exist a solution of model system to which is bounded in the region D'.

$$f_1 = \lambda - \xi - \zeta + \varpi C + \eta_1 C + \eta_2 I_F + \eta_3 I_M - \beta S - \epsilon_1 S - \epsilon 2S + \psi_1 R_D + \psi_2 R_N - \mu S \quad (14)$$

$$f_2 = \beta S - \eta_1 C + \xi - \kappa_1 C - \kappa_2 C - \mu C \quad (15)$$

$$f_3 = \kappa_1 C + \epsilon_1 S + \zeta - \eta_2 I_F - \rho_2 I_F - \rho_1 I_F - \alpha I_F - \mu I_F \quad (16)$$

$$f_4 = \kappa_2 C + \epsilon_2 S + \zeta(1-\zeta) - \eta_3 I_M - \rho_2 I_M - \rho_1 I_M - \alpha I_M - \mu I_M \quad (17)$$

$$f_5 = \rho_2 I_F + \rho_2 I_M - \psi_1 R_D - \delta R_D - \mu R_D \quad (18)$$

$$f_6 = \rho_1 I_F + \rho_1 I_M + \delta R_D - \psi_2 R_N - \mu R_N \quad (19)$$

It suffices to show that $\frac{\partial f_i}{\partial x_j}, i,j = 1,2,3,4,5,6$, are contin-

uous. Considering the partial derivatives

$$f_1 = \lambda - \xi - \zeta + \varpi C + \eta_1 C + \eta_2 I_F + \eta_3 I_M - \beta S - \epsilon_1 S - \epsilon 2S + \psi_1 R_D + \psi_2 R_N - \mu S$$

$$\left|\frac{\partial f_1}{\partial S}\right| = |-\xi - \zeta - \beta - \varepsilon_1 - \varepsilon_2 - \mu| < \infty,$$

$$\left|\frac{\partial f_1}{\partial C}\right| = |\varpi + \eta_1| < \infty,$$

$$\left|\frac{\partial f_1}{\partial I_F}\right| = |\eta_2| < \infty,$$

$$\left|\frac{\partial f_1}{\partial I_M}\right| = |\eta_3| < \infty,$$

$$\left|\frac{\partial f_1}{\partial R_D}\right| = |\psi_1| < \infty,$$

$$\left|\frac{\partial f_1}{\partial R_N}\right| = |\psi_2| < \infty.$$

Therefore

$$\left|\frac{\partial f_1}{\partial I_F}\right| = \left|\frac{\partial f_1}{\partial I_M}\right| = |0| < \infty,$$

$$f_2 = \beta S - \eta_1 C + \xi - \kappa_1 C - \kappa_2 C - \mu C,$$

$$\left|\frac{\partial f_2}{\partial S}\right| = |\xi| < \infty,$$

$$\left|\frac{\partial f_2}{\partial C}\right| = |-\eta_1 - \kappa_1 - \kappa_2 - \mu| < \infty,$$

$$\left|\frac{\partial f_2}{\partial I_F}\right| = |0| < \infty,$$

$$\left|\frac{\partial f_2}{\partial I_M}\right| = |0| < \infty,$$

$$\left|\frac{\partial f_2}{\partial R_D}\right| = |0| < \infty,$$

$$\left|\frac{\partial f_2}{\partial R_N}\right| = |0| < \infty.$$

Therefore

$$\left|\frac{\partial f_2}{\partial I_F}\right| = \left|\frac{\partial f_2}{\partial I_M}\right| = \left|\frac{\partial f_2}{\partial R_D}\right| = \left|\frac{\partial f_2}{\partial R_N}\right| = |0| < \infty,$$

$$f_3 = \kappa_1 C + \epsilon_1 S + \zeta - \eta_2 I_F - \rho_2 I_F - \rho_1 I_F - \alpha I_F - \mu I_F$$

$$\left|\frac{\partial f_3}{\partial S}\right| = |\varepsilon_1 + \zeta| < \infty,$$

$$\left|\frac{\partial f_3}{\partial C}\right| = |\kappa_1| < \infty,$$

$$\left|\frac{\partial f_3}{\partial I_F}\right| = |-\rho_2 - \rho_1 - \eta_2 - \alpha - \mu| < \infty,$$

$$\left|\frac{\partial f_3}{\partial I_M}\right| = |0| < \infty,$$

$$\left|\frac{\partial f_3}{\partial R_D}\right| = |0| < \infty,$$

$$\left|\frac{\partial f_3}{\partial R_N}\right| = |0| < \infty.$$

Therefore

$$\left|\frac{\partial f_3}{\partial S}\right| = \left|\frac{\partial f_3}{\partial I_M}\right| = \left|\frac{\partial f_3}{\partial R_D}\right| = \left|\frac{\partial f_3}{\partial R_N}\right| = |0| < \infty,$$

$$f_4 = \kappa_2 C + \epsilon_2 S + \zeta(1-\zeta) - \eta_3 I_M - \rho_2 I_M - \rho_1 I_M - \alpha I_M - \mu I_M$$

$$\left|\frac{\partial f_4}{\partial S}\right| = |\varepsilon_2 + \zeta(1-\zeta)| < \infty,$$

$$\left|\frac{\partial f_4}{\partial C}\right| = |\kappa_2| < \infty,$$

$$\left|\frac{\partial f_4}{\partial I_F}\right| = |0| < \infty,$$

$$\left|\frac{\partial f_4}{\partial I_M}\right| = |\rho_2 - \rho_1 - \eta_3 - \alpha - \mu| < \infty,$$

$$\left|\frac{\partial f_4}{\partial R_D}\right| = |0| < \infty,$$

$$\left|\frac{\partial f_4}{\partial R_N}\right| = |0| < \infty.$$

Therefore

$$\left|\frac{\partial f_4}{\partial S}\right| = \left|\frac{\partial f_4}{\partial I_F}\right| = \left|\frac{\partial f_4}{\partial R_D}\right| = \left|\frac{\partial f_4}{\partial R_N}\right| = |0| < \infty,$$

$$f_5 = \rho_2 I_F + \rho_2 I_M - \psi_1 R_D - \delta R_D - \mu R_D$$

$$\left|\frac{\partial f_5}{\partial S}\right| = |0| < \infty,$$

$$\left|\frac{\partial f_5}{\partial C}\right| = |0| < \infty,$$

$$\left|\frac{\partial f_5}{\partial I_F}\right| = |\rho_2| < \infty,$$

$$\left|\frac{\partial f_5}{\partial I_M}\right| = |\rho_2| < \infty,$$

$$\left|\frac{\partial f_5}{\partial R_D}\right| = |-\psi_1 - \delta - \mu| < \infty,$$

$$\left|\frac{\partial f_5}{\partial R_N}\right| = |0| < \infty.$$

Therefore

$$\left|\frac{\partial f_5}{\partial S}\right| = \left|\frac{\partial f_5}{\partial C}\right| = \left|\frac{\partial f_5}{\partial R_N}\right| = |0| < \infty,$$

$$f_6 = \rho_1 I_F + \rho_1 I_M + \delta R_D - \psi_2 R_N - \mu R_N$$

$$\left|\frac{\partial f_6}{\partial S}\right| = |0| < \infty,$$

$$\left|\frac{\partial f_6}{\partial C}\right| = |0| < \infty,$$

$$\left|\frac{\partial f_6}{\partial I_F}\right| = |\rho_1| < \infty,$$

$$\left|\frac{\partial f_6}{\partial I_M}\right| = |\rho_1| < \infty,$$

$$\left|\frac{\partial f_6}{\partial R_D}\right| = |\delta| < \infty,$$

$$\left|\frac{\partial f_6}{\partial R_N}\right| = |-\psi_2 - \mu| < \infty.$$

Therefore

$$\left|\frac{\partial f_6}{\partial S}\right| = \left|\frac{\partial f_6}{\partial C}\right| = |0| < \infty,$$

Clearly, all these partial derivatives are continuous and bounded, hence, by theorem 2, there exist a unique solution of Eq. (1) in the region D'.

8. Model Analysis

The first in the analysis is finding equilibria $(S^*, C^*, I_F^*, I_M^*, R_D^*, R_N^*)$ from the following equations

$$0 = \lambda - \xi - \zeta + \varpi C + \eta_1 C + \eta_2 I_F + \eta_3 I_M - \beta S - \epsilon_1 S - \epsilon 2S + \psi_1 R_D + \psi_2 R_N - \mu S \quad (20)$$

$$0 = \beta S - \eta_1 C + \xi - \kappa_1 C - \kappa_2 C - \mu C \quad (21)$$

$$0 = \kappa_1 C + \epsilon_1 S + \zeta - \eta_2 I_F - \rho_2 I_F - \rho_1 I_F - \alpha I_F - \mu I_F \quad (22)$$

$$0 = \kappa_2 C + \epsilon_2 S + \zeta(1-\zeta) - \eta_3 I_M - \rho_2 I_M - \rho_1 I_M - \alpha I_M - \mu I_M \quad (23)$$

$$0 = \rho_2 I_F + \rho_2 I_M - \psi_1 R_D - \delta R_D - \mu R_D \quad (24)$$

$$0 = \rho_1 I_F + \rho_1 I_M + \delta R_D - \psi_2 R_N - \mu R_N \quad (25)$$

The model Eq. (1) always has a disease free equilibrium $(DFE) P_0 = \frac{\lambda}{\mu}, 0, 0, 0, 0, 0)$. An endemic equilibrium $E^*(S^*, C^*, I_F^*, I_M^*, R_D^*, R_N^*$, which satisfies S^*, C^*, I_F^*, I_M^*, R_D^*, $R_N^* > 0$, from the equilibrium equation. It can be shown that a unique P^* exist with $S = \frac{\lambda+\varpi+\eta_1+\eta_2+\eta_3+\psi_1+\psi_2}{\xi+\zeta+\beta+\varepsilon_1+\varepsilon_2+\mu}$ for P^* to exist in the feasible region Ω and necessary sufficient that $0 < S^* \leq \frac{\lambda}{\mu}$ or alternatively $\frac{\lambda}{\mu S^*} \geq 1$.

Define, $R_0 = \frac{1}{S^*} * \frac{\lambda}{\alpha}$, then R_0 is a threshold parameter that used to determine the number of equilibria.

Theorem 3: If $R_0 > 1$, then P^* is globally asymptotically stable with respect to the interior of Ω.

Combination of these two programs (education & vaccination) effect the possible outbreak of meningitis eg. When the rate of educational campaign = 10% with boost the rate of vaccination = 20%. Considering normal carrier rate which is 25% and the proportion of 21% must be vaccinated to eliminate meningitis infection. Therefore,

$$S^* = \frac{\lambda+\varpi+\eta_1+\eta_2+\eta_3+\psi_1+\psi_2}{\xi+\zeta+\beta+\varepsilon_1+\varepsilon_2+\mu}$$

$$\frac{0.41+0.25+0.4+0.3+0.3+0.3+0.015+0.045}{0.25(0.1+0.02+0.4+0.052+0.02}$$

$$= \frac{1.75}{0.808}$$

$$= 2.17$$

$$R_0 = \frac{1}{S} * \frac{\lambda}{\mu}$$

$$= \frac{1}{2.17} * \frac{0.41}{0.04}$$

$$= \frac{0.41}{0.0434}$$

$$= 9.4$$

Therefore $R_0 = 9.4 > unity$.

Since $R_0 > 1$, therefore the disease will persist therefore, It is observed from the threshold that the combination of an education program coupled with a vaccination program would greatly reduce the number of infected individuals and carriers in the event of an outbreak.

9. Stability of a Disease Free Equilibrium

To understand how the parameters affect the meningitis model, the stability nature of the Disease Free Equilibrium is analyzed by finding the Jacobian matrix for the S, C, I_F, I_M, R_D, R_N system. Jacobian matrix is used in order to determine the local stability of the disease free equilibrium $P_0 = (\frac{\lambda}{\mu}, 0, 0, 0, 0, 0)$.

Evaluation of the local stability of the disease-free-equilibrium P_0 by jacobian matrix

The evaluation follows Jacobian matrix is used in order to determine the local stability of the disease free equilibrium $P_0 = \left(\frac{\lambda}{\alpha}, 0, 0, 0, 0, 0\right)$.

$$J(P_0) = \begin{pmatrix} A & \varpi+\eta_1 & \eta_2 & \eta_3 & \psi_1 & \psi_2 \\ \beta+\xi & B & 0 & 0 & 0 & 0 \\ \varepsilon_1+\zeta & \kappa_1 & C & 0 & 0 & 0 \\ \varepsilon_2+\zeta(1-\zeta) & \kappa_2 & 0 & D & 0 & 0 \\ 0 & 0 & \rho_2 & \rho_2 & E & 0 \\ 0 & 0 & \rho_1 & \rho_1 & \delta & F \end{pmatrix} \quad (26)$$

where

$$A = -\beta - \varepsilon_1 - \varepsilon_2 - \xi - \mu,$$
$$B = -\eta_1 - \kappa_1 - \kappa_2 - \mu,$$
$$C = -\eta_2 - \rho_2 - \rho_1 - \alpha - \mu,$$
$$D = -\eta_3 - \rho_1 - \rho_2 - \alpha - \mu,$$
$$E = -\delta - \psi_1 - \mu,$$
$$F = -\psi_2 - \mu$$

This is the stability result which shows that the model is locally asymptotically stable.

Proposition 1: P_0 is asymptotically stable if $R_e(\lambda) < 0$.

Proof: $|J(P_0 - \lambda I)|$

$$J(P_0) = \begin{vmatrix} -\mu-\lambda & \varpi & 0 & 0 & 0 & 0 \\ 0 & -\mu-\lambda & 0 & 0 & 0 & 0 \\ \zeta & 0 & -\alpha-\mu-\lambda & 0 & 0 & 0 \\ \zeta(1-\zeta) & 0 & 0 & -\alpha-\mu-\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & -\mu-\lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mu-\lambda \end{vmatrix} \tag{27}$$

The eigenvalues of $J(P_0)$ are

$$\lambda_1 = -\mu$$
$$\lambda_2 = -\mu$$
$$\lambda_3 = -\alpha - \mu$$
$$\lambda_4 = -\alpha - \mu$$
$$\lambda_5 = -\mu$$
$$\lambda_6 = -\mu$$

Therefore, $R_e\lambda < 0$ since all the parameters are non negative. This proves the preposition. □

10. Numerical Simulation

Numerical simulation of model is carried out. Epidemiological model for the control of transmission dynamics of meningitis infection is presented incorporated with control parameters such as educational campaign and vaccination. The standard ordinary differential equation of the model is used to predict the dynamics and the control of the disease. Matrix Laboratory (Matlab R2014a) application software was use to solve the system of equations in the model. Numerical simulations of the model were done, as well as the plots of the graphs. This was done to look into the effects and changes that will occur in the model as the values of each of the compartments of the model were used. The existence and stability of the disease- free equilibrium and endemic equilibrium states of the model equations were computed. Some values for the model parameters for the components of the model were based on specified values as being used in the models by references and some values were Estimated.

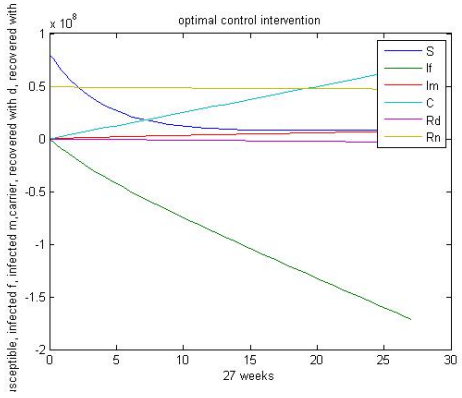

Figure 2. Graphical representation of Meningitis Transmission with control parameters

11. Conclusion

In this study, the control intervention parameters are included represented by educational campaign and vaccination modelled on the transmission dynamics of meningitis infections. The analysis of the Endemic equilibrium state of the model, using the threshold value, $R_0 > 1$ shows that meningitis infections cannot be controlled effectively or eradicated with when normal carrier rate is considered at 25% since the calculated threshold value R_0 is 9.4 which is greater than 1, meaning that the disease cannot be controlled without using control variables represented by educational campaign and vaccination. The control with educational campaign and vaccination for Meningitis transmission model with carrier has been studied. The numerical results show the effectiveness of control on Meningitis epidemics.

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