Solving system of DAEs by Modified Homotopy Perturbation Method

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Abstract: This paper presents using a new modified homotopy perturbation method (NHPM) for solving linear and nonlinear systems of differential-algebraic equations (DAEs) of index one or higher index without index reduction. By using this scheme, explicit exact solution is calculated in the form of a convergent power series with easily computable components. Some examples are given to illustrate the simplicity and reliability of the new method. The obtained results are found to be in good agreement with the exact solutions known.

Keywords: Differential algebraic equations, New Homotopy perturbation method.

1. Introduction

Differential Algebraic Equations (DAEs) can be found in a wide variety of scientific and engineering applications including circuit analysis, computer-aided design and real-time simulation of mechanical systems, power systems, chemical process simulation [1], and optimal control.

Differential Algebraic Equations (DAEs) are system of differential equations, where the unknown functions to satisfy additional algebraic equations. In other words, they consist of a set of differential equations with additional algebraic constraints.

In recent years, a great deal of research has been focused on the numerical solution of systems of DAEs [2-4]. The first practical numerical methods for certain classes of DAEs are the Backward Differentiation Formula (BDF) [12] and implicit Runge-Kutta methods [13]. Recently Adomian Decomposition Method (ADM) [5, 6], Variational Iterational Method (VIM) [7] pseudo-spectral method [14], and Homotopy Perturbation Method (HPM) [8, 9] have been used to solve the linear and nonlinear DAEs.

The first homotopy perturbation method is proposed by Dr. Ji Huan He [16, 17]. This method is useful for obtaining exact and approximate solutions of linear and nonlinear differential equations. This method, which is a combination of homotopy in topology and the classic perturbation techniques, provides us with a convenient way to obtain analytic or approximate solutions for a wide variety of problems arising in different fields. On the other hand, the homotopy methods have been proved to be globally convergent for a general class of nonlinear equations; therefore they are useful for solving nonlinear equations. Dr He used HPM to solve nonlinear wave equations [18], boundary value problems [19, 20], quadratic Riccati differential equations [21], integral equations [22-24], initial value problems [25, 26], differential-difference equations [27], modified KdV equation [28] and many other problems [29-31]. This wide variety of applications shows the effectiveness of HPM in solving functional equations.

In this paper, we illustrate the ability of the new form of the Homotopy Perturbation Method (NHPM) introduced by Aminikhah and Hemmatnezhad [10], on index-1 and higher index DAEs. The numerical results show that NHPM technique gives the approximate solution with higher accuracy compared to the results produced by HPM. This numerical scheme is based on Taylor series expansion and is capable of finding the exact solution of many nonlinear differential equations. NHPM has successfully applied to stiff systems of ODEs [10], initial-type differential equations of heat transfer, nonlinear strongly differential equations and stiff delay differential equations (DDEs) [15]. The rest of this paper is organized as following: at first we've described the DAEs then we've discussed the NHPM. After that we've used two examples to show the effectiveness of the method and finally the conclusion is drawn.

2. Differential Algebraic Equations

Differential algebraic equations (DAEs) are in the form of

$$f(X', X, y, t) = 0 \tag{1}$$

where $X \in \Re^n$ is the vector of differential variables, $y \in \Re^m$ is the vector of algebraic variables, $t \in \Re$ is the independent variable, and $f: \Re^{2n+m+1} \to \Re^{n+m}$ is the set of DAEs [6].

The index of DAEs is a measure of the degree of singularity of the system and also widely regarded as an indication of certain difficulties for numerical methods. The DAEs with higher index, i.e., an index greater than one, is in a sense ill posed. The index-1 semi-explicit DAEs is given by:

$$\begin{cases} X'(t) = f(X(t), y(t), t) \\ 0 = g(X(t), y(t), t) \end{cases} \tag{2}$$

where $\frac{\partial g}{\partial y}$ is nonsingular [11].

Here, we are going to present implementation of New Homotopy Perturbation Method for a system of DAEs. To do that, first of all we explain New Homotopy Perturbation Method to solve a nonlinear ODE.

3. New Homotopy Perturbation Method

To illustrate the basic ideas of this method, we consider the following nonlinear differential equation:

$$A(\mathbf{u}) - f(t) = 0, \quad \mathbf{u}(0) = \mathbf{u}_0, \quad t \in \Omega \tag{3}$$

where A is a general differential operator, $\mathbf{u}_0$ is an initial approximation of Eq. (3), and $f(t)$ is a known analytical function on the domain of Ω. The operator A can be divided into two parts, which are L and N, where L is a linear operator, but N is nonlinear. Eq. (3) can be, therefore, rewritten as follows:

$$L(\mathbf{u}) + N(\mathbf{u}) - f(t) = 0$$

By the homotopy technique, we construct a homotopy $\mathbf{U}(t,p):\Omega\times[0,1]\to\Re$, which satisfies:

$$\mathrm{H}(\mathbf{U},\mathrm{p}) = (1-\mathrm{p})[\mathrm{L}\mathbf{U}(\mathrm{t}) - \mathrm{L}\mathbf{u}_0(\mathrm{t})] + \mathrm{p}[\mathrm{A}\mathbf{U}(\mathrm{t}) - \mathrm{f}(\mathrm{t})] = 0, \quad \mathrm{p}\in[0,1], \mathrm{t}\in\Omega \tag{4}$$

or

$$\mathrm{H}(\mathbf{U},\mathrm{p}) = \mathrm{L}\mathbf{U}(\mathrm{t}) - \mathrm{L}\mathbf{u}_0(\mathrm{t}) + \mathrm{p}\mathrm{L}\mathbf{u}_0(\mathrm{t}) + \mathrm{p}[\mathrm{N}\mathbf{U}(\mathrm{t}) - \mathrm{f}(\mathrm{t})] = 0, \quad \mathrm{p}\in[0,1], \mathrm{t}\in\Omega \tag{5}$$

where $\mathrm{p}\in[0,1]$ is an embedding parameter, which satisfies the boundary conditions. Obviously, from Eqs. (4) or (5) we will have

$$\mathrm{H}(\mathbf{U},0) = \mathrm{L}\mathbf{U}(\mathrm{t}) - \mathrm{L}\mathbf{u}_0(\mathrm{t}) = 0, \quad \mathrm{H}(\mathbf{U},1) = \mathrm{A}\mathbf{U}(\mathrm{t}) - \mathrm{f}(\mathrm{t}) = 0 \tag{6}$$

The changing process of p from zero to unity is just that of $\mathbf{U}(t,p)$ from $\mathbf{u}_0(\mathrm{t})$ to $\mathbf{u}(t)$. In topology, this is called homotopy. According to the HPM, we can first use the embedding parameter p as a small parameter, and assume that the solution of Eqs. (4) or (5) can be written as a power series in p:

$$\mathbf{U} = \sum_{n=0}^{\infty} \mathrm{p}^n \mathbf{U}_n = \mathbf{U}_0 + \mathrm{p}\mathbf{U}_1 + \mathrm{p}^2\mathbf{U}_2 + \mathrm{p}^3\mathbf{U}_3 + \cdots \tag{7}$$

Setting $p = 1$, results in the approximate solution of Eq. (3)

$$\mathbf{u}(\mathrm{t}) = \lim_{\mathrm{p}\to 1}\mathbf{U} = \mathbf{U}_0 + \mathbf{U}_1 + \mathbf{U}_2 + \mathbf{U}_3 + \cdots \tag{8}$$

Applying the inverse operator $\mathrm{L}^{-1} = \int_0^t (\cdot)\mathrm{dt}$ to both sides of Eq. (5), we obtain

$$\mathbf{U}(\mathrm{t}) = \mathbf{U}(0) + \int_0^t \mathrm{L}\mathbf{u}_0(\mathrm{t})\mathrm{dt} - \mathrm{p}\int_0^t \mathrm{L}\mathbf{u}_0(\mathrm{t})\mathrm{dt} - \mathrm{p}\left[\int_0^t (\mathrm{N}\mathbf{U}(\mathrm{t}) - \mathrm{f}(\mathrm{t}))\mathrm{dt}\right] \tag{9}$$

where $\mathbf{U}(0) = \mathbf{u}_0$.

Now, suppose that the initial approximations to the solutions, $\mathrm{L}\mathbf{u}_0(\mathrm{t})$, have the form

$$\mathrm{L}\mathbf{u}_0(\mathrm{t}) = \sum_{n=0}^{\infty} \boldsymbol{\alpha}_n \mathrm{P}_n(\mathrm{t}) \tag{10}$$

where $\boldsymbol{\alpha}_n$ are unknown coefficients, and $\mathrm{P}_0(\mathrm{t}), \mathrm{P}_1(\mathrm{t}), \mathrm{P}_2(\mathrm{t}), \ldots$ are specific functions.

Substituting (7) and (10) into (9) and equating the coefficients of p with the same power leads to

$$\begin{cases} \mathrm{p}^0: \mathbf{U}_0(\mathrm{t}) = \mathbf{u}_0 + \sum_{n=0}^{\infty} \boldsymbol{\alpha}_n \int_0^t \mathrm{P}_n(t)dt \\ \mathrm{p}^1: \mathbf{U}_1(\mathrm{t}) = -\sum_{n=0}^{\infty} \boldsymbol{\alpha}_n \int_0^t \mathrm{P}_n(t)dt - \int_0^t (N\mathbf{U}_0(t) - f(t))dt \\ \mathrm{p}^2: \mathbf{U}_2(\mathrm{t}) = -\int_0^t N\mathbf{U}_1(t)dt \\ \mathrm{p}^j: \mathbf{U}_j(\mathrm{t}) = -\int_0^t N\mathbf{U}_{j-1}(t)dt \end{cases} \tag{11}$$

Now, if these equations are solved in such a way that $\mathbf{U}_1(\mathrm{t}) = 0$, then Eq. (11) results in

$\mathbf{U}_1(\mathrm{t}) = \mathbf{U}_2(\mathrm{t}) = \mathbf{U}_3(\mathrm{t}) = \cdots = 0$.

and therefore the exact solution can be obtained by using

$$\mathbf{U}(\mathrm{t}) = \mathbf{U}_0(\mathrm{t}) = \mathbf{u}_0 + \sum_{n=0}^{\infty} \boldsymbol{\alpha}_n \int_0^t \mathrm{P}_n(t)dt \tag{12}$$

It is worth noting that, if $\mathrm{U}(\mathrm{t})$ is analytic at $\mathrm{t} = \mathrm{t}_0$, then their Taylor series

$$\mathbf{U}(\mathrm{t}) = \sum_{n=0}^{\infty} \mathbf{a}_n (\mathrm{t} - \mathrm{t}_0)^n$$

can be used in Eq. (11), where $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2, \ldots$ are known coefficients and $\boldsymbol{\alpha}_n$ are unknown ones, which must be computed.

We explain this method by considering two examples in the following.

4. Test Problems

4.1 Example 1.

Consider linear DAEs

$$\begin{aligned} X' &= AX + BY + q \\ 0 &= CX + r \end{aligned} \tag{13}$$

for $0 \le t \le 1$, with $x_1(0) = x_2(0) = x_3(0) = 0$,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & t \end{bmatrix}, \quad C^T = B = \begin{bmatrix} 1 & -1 \\ 0 & t \\ -1 & 0 \end{bmatrix},$$

q and r are compatible to exact solutions $x_1(t) = x_2(t) = x_3(t) = t^4 + t^5$ and $y_1(t) = y_2(t) = t/(t^4 + t^5 + 1)$. According to Theorem 2, presented in [5], index-2 problem (13) can be transformed to index-1 DAE:

$$\begin{cases} x_1 = tx_2 - g_1(t) \\ x_2' = g_2(t) - tx_1' - tx_3' + tx_1 + x_2 + t^2 x_3 \\ x_3 = x_1 \end{cases} \tag{14}$$

where

$g_1(t) = t^6 - t^4$, $g_2(t) = 4t^3 + 12t^4 + 8t^5 - 2t^6 - t^7$.

For solving system (14) by NHPM, we construct the following homotopy:

$$\begin{cases}(1-p)\left[U_1(t)-u_{1,0}(t)\right]+p\left[U_1(t)-tU_2+g_1(t)\right]=0\\(1-p)\left[LU_2(t)-Lu_{2,0}(t)\right]+\\\qquad p\left[LU_2(t)-g_2(t)+tU_1'+tU_3'-tU_1-U_2-t^2U_3\right]=0\\U_3=U_1\end{cases}\quad(15)$$

or

$$\begin{cases}U_1(t)-u_{1,0}(t)+pu_{1,0}(t)+p\left[-tU_2+g_1(t)\right]=0\\LU_2(t)-Lu_{2,0}(t)+pLu_{2,0}(t)+\\\qquad p\left[-g_2(t)+2tU_1'-tU_1-U_2-t^2U_1\right]=0\end{cases}\quad(16)$$

where $L=\dfrac{d}{dt}$ and $p\in[0,1]$ is an embedding parameter.

Assume that

$$u_{1,0}(t)=\sum_{n=0}^{\infty}\alpha_n P_n(t),$$

$$Lu_{2,0}(t)=\sum_{n=0}^{\infty}\beta_n P_n(t),\qquad P_n(t)=t^n \quad(17)$$

and from the initial conditions

$$U_2(0)=0,$$

Substituting (17) into (16) and applying the inverse operator $L^{-1}=\int_0^t(\cdot)dt$ to Eq. (16), we have

$$\begin{cases}U_1(t)=\sum_{n=0}^{\infty}\alpha_n t^n-p\sum_{n=0}^{\infty}\alpha_n t^n-p(-tU_2+g_1(t)),\\U_2(t)=U_2(0)+\sum_{n=0}^{\infty}\frac{\beta_n}{n+1}t^{n+1}-p\sum_{n=0}^{\infty}\frac{\beta_n}{n+1}t^{n+1}-\\\qquad p\int_0^t(-g_2(t)+2tU_1'(t)-tU_1(t)-U_2(t)-t^2U_1(t))dt,\end{cases}\quad(18)$$

Suppose the solutions of system (14) to be in the following form

$$U_i=\sum_{n=0}^{\infty}p^n U_{i,n}=U_{i,0}+pU_{i,1}+p^2U_{i,2}+p^3U_{i,3}+\cdots\quad i=1,2 \quad(19)$$

where in $U_{i,j}$ for i=1,2 and j=0,1,2,3,... are functions which should be determined.

Substituting (19) into (18) and equating the coefficients of p with the same powers leads to

$$p^0:\begin{cases}U_{1,0}(t)=\sum_{n=0}^{\infty}\alpha_n t^n=\sum_{n=0}^{\infty}\alpha_n t^n,\\U_{2,0}(t)=U_2(0)+\sum_{n=0}^{\infty}\frac{\beta_n}{n+1}t^{n+1}=\sum_{n=0}^{\infty}\frac{\beta_n}{n+1}t^{n+1}\end{cases}$$

$$p^1:\begin{cases}U_{1,1}(t)=-\sum_{n=0}^{\infty}\alpha_n t^n+(tU_{2,0}-g_1(t)),\\U_{2,1}(t)=-\sum_{n=0}^{\infty}\frac{\beta_n}{n+1}t^{n+1}+\\\qquad\int_0^t\left(g_2(t)-2tU'_{1,0}(t)+tU_{1,0}(t)+U_{2,0}(t)+t^2U_{1,0}(t)\right)dt,\end{cases}$$

$$p^j:\begin{cases}U_{1,j}(t)=tU_{2,j-1},\\U_{2,j}(t)=\int_0^t\left(-2tU'_{1,j-1}(t)+tU_{1,j-1}(t)+U_{2,j-1}(t)+t^2U_{1,j-1}(t)\right)dt,\end{cases}\quad j=2,3,\ldots$$

Now, if we set the Taylor series of $U_{1,1}(t)$ and $U_{2,1}(t)$ at $t=0$ equal to zero, leads to

$$\alpha_0=\alpha_1=\alpha_2=\alpha_3=0,\quad \alpha_4=\alpha_5=1,\qquad \alpha_6=\alpha_7=\alpha_8=\cdots=0$$

$$\beta_0=\beta_1=\beta_2=\beta_3=0,\quad \beta_4=4,\beta_5=5,\quad \beta_6=\beta_7=\beta_8=\cdots=0$$

Therefore, the approximate solutions of the system of differential equation (14) can be expressed as

$$x_1(t)=U_{1,0}(t)=t^4+t^5,$$

$$x_2(t)=U_{2,0}(t)=t^4+t^5,$$

$$x_3(t)=x_1(t)=t^4+t^5,$$

which are the exact solutions.

It is considerable that we can solve this problem directly and without index reduction. By similar manner we get

$$x_1(t)=U_{1,0}(t)=t^4+t^5,$$

$$x_2(t)=U_{2,0}(t)=t^4+t^5,$$

$$x_3(t)=U_{3,0}(t)=t^4+t^5,$$

$$y_1(t)=U_{4,0}(t)=t-t^5-t^6+t^9+2t^{10}+t^{11}-t^{13}-3t^{14}$$
$$\qquad -3t^{15}-t^{16}+t^{17}+\cdots$$

$$y_2(t)=U_{5,0}(t)=t-t^5-t^6+t^9+2t^{10}+t^{11}-t^{13}-3t^{14}$$
$$\qquad -3t^{15}-t^{16}+t^{17}+\cdots$$

where $U_{4,0}(t)$ and $U_{5,0}(t)$ are the Taylor expansion of the exact solutions.

This example shows that this method can solve higher index DAEs.

4.2 Example 2.

Consider nonlinear DAEs

$$\begin{cases}x_1'=x_1-x_2x_3+\sin(t)+t\cos(t)\\x_2'=tx_3+x_1^2+\sec^2(t)-t^2\left(\cos(t)+\sin^2(t)\right)\\0=x_1-x_3+t\left(\cos(t)-\sin(t)\right)\end{cases}\quad(20)$$

for $0\le t\le 1$, with initial conditions, $x_1(0)=x_2(0)=x_3(0)=0$, and exact solutions $x_1(t)=t\sin(t)$, $x_2(t)=\tan t$ and $x_3(t)=t\cos t$. According to (2), the above problem is an index-1 semi-explicit DAEs [6].

For solving system (20) by NHPM, we construct the following homotopy:

$$\begin{cases}(1-p)\left[LU_1(t)-Lu_{1,0}(t)\right]+p\left[LU_1(t)-U_1+U_2U_3-g_1(t)\right]=0\\(1-p)\left[LU_2(t)-Lu_{2,0}(t)\right]+p\left[LU_2(t)-tU_3-U_1^3-g_2(t)\right]=0\\(1-p)\left[U_3(t)-u_{3,0}(t)\right]+p\left[U_3(t)-U_1-g_3(t)\right]=0\end{cases}\quad(21)$$

or

$$\begin{cases}LU_1(t)-Lu_{1,0}(t)+pLu_{1,0}(t)+p\left[-U_1+U_2U_3-g_1(t)\right]=0\\LU_2(t)-Lu_{2,0}(t)+pLu_{2,0}(t)+p\left[-tU_3-U_1^3-g_2(t)\right]=0\\U_3(t)-u_{3,0}(t)+pu_{3,0}(t)+p\left[-U_1-g_3(t)\right]=0\end{cases}\quad(22)$$

where $L=\dfrac{d}{dt}$ and $p\in[0,1]$ is an embedding parameter and $g_1(t)$, $g_2(t)$ and $g_3(t)$ are Taylor expansions of

$\sin(t)+t\cos(t)$, $\sec^2(t)-t^2(\cos(t)+\sin^2(t))$ and $t(\cos(t)-\sin(t))$, respectively.

Assume that

$$Lu_{1,0}(t)=\sum_{n=0}^{\infty}\alpha_n P_n(t),\ Lu_{2,0}(t)=\sum_{n=0}^{\infty}\beta_n P_n(t),$$

$$u_{3,0}(t)=\sum_{n=0}^{\infty}\delta_n P_n(t),\qquad P_n(t)=t^n \tag{23}$$

and from the initial conditions

$$U_1(0)=0, U_2(0)=0, U_3(0)=0$$

Substituting (23) into (22) and applying the inverse operator $L^{-1}=\int_0^t(\cdot)dt$ to Eq. (22), we have

$$\begin{cases} U_1(t)=U_1(0)+\sum_{n=0}^{\infty}\frac{\alpha_n}{n+1}t^{n+1}-p\sum_{n=0}^{\infty}\frac{\alpha_n}{n+1}t^{n+1}-p\int_0^t\left(-U_1+U_2U_3-g_1(t)\right)dt,\\ U_2(t)=U_2(0)+\sum_{n=0}^{\infty}\frac{\beta_n}{n+1}t^{n+1}-p\sum_{n=0}^{\infty}\frac{\beta_n}{n+1}t^{n+1}-p\int_0^t\left(-tU_3-U_1^3-g_2(t)\right)dt,\\ U_3(t)=\sum_{n=0}^{\infty}\delta_n t^n-p\sum_{n=0}^{\infty}\delta_n t^n-p(-U_1-g_3(t)) \end{cases}$$

Suppose the solutions of system (20) to be in the following form

(24)

$$U_i=\sum_{n=0}^{\infty}p^n U_{i,n}=U_{i,0}+pU_{i,1}+p^2U_{i,2}+p^3U_{i,3}+\cdots\quad i=1,2 \tag{25}$$

where in $U_{i,j}$ for i=1,2,3 and j=0,1,2,3,... are functions which should be determined.

Substituting (25) into (24) and equating the coefficients of p with the same powers leads to

$$p^0:\begin{cases} U_{1,0}(t)=U_1(0)+\sum_{n=0}^{\infty}\frac{\alpha_n}{n+1}t^{n+1}=\sum_{n=0}^{\infty}\frac{\alpha_n}{n+1}t^{n+1},\\ U_{2,0}(t)=U_2(0)+\sum_{n=0}^{\infty}\frac{\beta_n}{n+1}t^{n+1}=\sum_{n=0}^{\infty}\frac{\beta_n}{n+1}t^{n+1},\\ U_{3,0}(t)=\sum_{n=0}^{\infty}\delta_n t^n=\sum_{n=0}^{\infty}\delta_n t^n \end{cases}$$

$$p^1:\begin{cases} U_{1,1}(t)=-\sum_{n=0}^{\infty}\frac{\alpha_n}{n+1}t^{n+1}+\int_0^t\left(U_{1,0}-U_{2,0}U_{3,0}+g_1(t)\right)dt,\\ U_{2,1}(t)=-\sum_{n=0}^{\infty}\frac{\beta_n}{n+1}t^{n+1}+\int_0^t\left(tU_{3,0}+U_{1,0}^{\ 3}+g_2(t)\right)dt,\\ U_{3,1}(t)=-\sum_{n=0}^{\infty}\delta_n t^n+\left(U_1+g_3(t)\right) \end{cases}$$

$$p^j:\begin{cases} U_{1,j}(t)=\int_0^t\left(U_{1,j-1}-U_{2,j-1}U_{3,j-1}\right)dt,\\ U_{2,j}(t)=\int_0^t\left(tU_{3,j-1}+U_{1,j-1}^{\ 3}\right)dt, & j=2,3,\ldots\\ U_{3,j}(t)=U_{1,j-1} \end{cases}$$

Now, if we set the Taylor series of $U_{i,1}(t), i=1,2,3$ at $t=0$ equal to zero, unknown coefficients $\alpha_j,\beta_j,\delta_j, j=0,1,2,\ldots$ will be determined.

Therefore, the approximate solutions of the system of differential equation (20) can be expressed as

$$x_1(t)=U_{1,0}(t)=t^2-\frac{t^4}{3!}+\frac{t^6}{5!}-\frac{t^8}{7!}+-\cdots$$
$$=t\left(t^1-\frac{t^3}{3!}+\frac{t^5}{5!}-\frac{t^7}{7!}+-\cdots\right)=t\sin t$$

$$x_2(t)=U_{2,0}(t)=t-\frac{t^3}{3}+\frac{2t^5}{15}+\frac{17t^7}{315}+\frac{62t^7}{2835}+\cdots=\tan t$$

$$x_3(t)=U_{3,0}(t)=t-\frac{t^3}{2!}+\frac{t^5}{4!}-\frac{t^7}{6!}+-\cdots$$
$$=t\left(1-\frac{t^2}{2!}+\frac{t^4}{4!}-\frac{t^6}{6!}+-\cdots\right)=t\cos t$$

5. Conclusion

In this work, the new homotopy perturbation method has been successfully applied for solving linear and nonlinear differential algebraic equations. Examples 1 and 2 show that we can solve higher index DAEs without index reduction and achieve a very good approximation to the actual solution of the equations by using only one iteration of the NHPM. As we can see this method will be useful for a system of differential equations.

Since the real world problems lead to the solution of nonlinear equations or systems of nonlinear equations, it would be very interesting to extend this method to such problems. Research in this matter is one of our future goals.

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