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Characterizations on Decreasing Laplace Transform of Time to Failure Class and Hypotheses Testing

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Abstract: In this paper, we investigate the probabilistic characteristics for Decreasing Laplace Transform of Time to Failure ($DLTTF$ class). The closure properties under various reliability operations such as convolution, mixture and the homogeneous Poisson shock model are studied. A new hypothesis test is constructed to test exponentiality against $DLTTF$ based on goodness of fit approach. Pitman asymptotic efficiency PAE and Pitman asymptotic relative efficiency $PARE$ are studied. The critical values of the test are calculated and tabulated, and the power estimates are calculated to assess the performance of the test. Finally, sets of real data are used as examples to elucidate the use of the proposed test statistic for practical problems in the reliability analysis.

Keywords: convolution, mixture, Homogeneous Poisson shock model, hypothesis test, efficiency, monte Carlo method, power.

1. Introduction

In reliability theory, positive aging describes the situation where the time to failure tends to decrease, in some probabilistic sense, with increasing age, that is, the age has an adverse effect on the time to failure lifetime. Negative aging describes the opposite beneficial effect. If the same type of aging persists throughout the entire life of a unit, it is called monotonic aging. However, in many practical situations, the effect of age is initially beneficial, where negative aging takes place but, after a certain period, the effect of age is adverse and the aging is positive. This kind of non-monotonic aging arises naturally in situations like infant mortality, work hardening of mechanical or electronic machines, and lengths of political coalitions or divorce rates.

Failure of a unit during actual operation is costly or dangerous. If the unit is characterized by a failure rate that increases with age, it may be reasonable to replace it before it has aged too greatly. A commonly considered replacement policy is the policy based on age which is in force if a unit is always replaced at the time of failure or t hours after its installation, whichever comes first.

As a useful notion in applied mathematics and engineering, Laplace transform is very important in many areas of probability and statistics [1]. For two non-negative random variables X and Y with distribution functions F and G, (survival functions $\overline{F}$, $\overline{G}$) respectively, then X is smaller than Y in Laplace transform order (denoted by $X \leq_{LT} Y$) if, and only, if

$$\int_0^\infty e^{-sx}\overline{F}(x)\,dx \leq \int_0^\infty e^{-sy}\overline{G}(y)\,dy, \text{ for all } s \geq 0$$

Definition 1 ([2]): The lifetime random variable X is said to be decreasing Laplace transform of time to failure ($DLTTF$) class if, and only if,

$$f(t)\int_0^t e^{-sx}\overline{F}(x)\,dx \geq \overline{F}(t)\int_0^t e^{-sx}f(x)\,dx,\ t>0, s\geq 0. \quad (1)$$

On the other hand, statisticians and reliability analysts have shown a growing interest in modeling survival data using classifications of life distributions. These categories are useful for modeling situations, maintenance, inventory theory, and biometry (cf. [3, 4]).

The construction of this paper is as follows: In Section 2, we discusse preservation under convolution, mixture, and the homogeneous Poisson shock model for $DLTTF$ class of life distribution. In section 3, we present testing exponentiality against $DLTTF$ class. The Pitman asymptotic efficiency (PAE) and Pitman asymptotic relative efficiency ($PARE$) are calculated for some commonly used distributions in reliability in Section 4. In Section 5, Monte Carlo null distribution critical points are simulated for sample sizes $n = 5(5)50$ and the power estimates of the tests are also calculated. Finally, in Section 6, we discuss some applications to elucidate the usefulness of the proposed tests in reliability analysis.

2. Closure Properties

In this section, we study the closure properties of $DLTTF$ class under some reliability operations such as convolution, mixture and the shock model in homogeneous case.

2.1 Convolution properties

The aim of this subsection is to discuss preservation under convolution properties of $DLTTF$ class.

Theorem 1: The $DLTTF$ class is preserved under convolution.

Proof: Suppose that f_1, F_1 and f_2, F_2 are independent $DLTTF$ life time distributions and their convolution is given by:

$$f(x)=\int_0^\infty f_1(x-y)\,f_2(y)\,dy,$$
$$\overline{F}(x)=\int_0^\infty \overline{F}_1(x-y)\,dF_2(y).$$

then,

$$f(t)\int_0^t e^{-sx}\overline{F}(x)\,dx=\left[\int_0^\infty f_1(t-y)\,f_2(y)\,dy\right]$$
$$\cdot\left[\int_0^t e^{-sx}\left(\int_0^\infty \overline{F}_1(x-y)\,dF_2(y)\right)dx\right]$$
$$=\int_0^\infty\int_0^\infty f_2(y)\left[f_1(t-y)\int_0^t e^{-sx}\overline{F}_1(x-y)\,dx\right]dF_2(y)\,dy.$$

Since $f_1, \overline{F}_1$ is $DLTTF$ then

$$f(t)\int_0^t e^{-sx}\overline{F}(x)\,dx$$
$$\geq\int_0^\infty\int_0^\infty f_2(y)\left[\overline{F}_1(t-y)\int_0^t e^{-sx}f_1(x-y)\,dx\right]dF_2(y)\,dy$$
$$\geq\overline{F}(t)\int_0^t e^{-sx}f(x)\,dx.$$

Which complete the proof. □

2.2 Mixture properties

The following theorem is stated and proved to show that the $DLTTF$ class is preserved under mixture.

Theorem 2: The $DLTTF$ class is preserved under mixture.

Proof: Suppose that $f(x)$, $F(x)$ is the mixture of f_α and F_α, where all are $DLTTF$ since

$$f(x)=\int_0^\infty f_\alpha(x)\,dg(\alpha),$$
$$\overline{F}(x)=\int_0^\infty \overline{F}_\alpha(x)\,dG(\alpha),$$

then,

$$f(t)\int_0^t e^{-sx}\overline{F}(x)\,dx=\left[\int_0^\infty f_\alpha(t)\,dg(\alpha)\right]$$
$$\cdot\int_0^t e^{-sx}\left[\int_0^\infty \overline{F}_\alpha(x)\,dG(\alpha)\right]dx$$
$$=\int_0^\infty\int_0^\infty f_\alpha(t)\int_0^t e^{-sx}\overline{F}_\alpha(x)\,dx\,dG(\alpha)\,dg(\alpha),$$

since f_α and F_α is $DLTTF$ then,

$$f(t)\int_0^t e^{-sx}\overline{F}(x)\,dx\geq$$
$$\int_0^\infty\int_0^\infty \overline{F}_\alpha(t)\int_0^t e^{-sx}f_\alpha(x)\,dx\,dg(\alpha)\,dG(\alpha)$$

Upon using Chebyschev inequality for similarity ordered functions we get,

$$f(t)\int_0^t e^{-sx}\overline{F}(x)\,dx\geq\int_0^\infty \overline{F}_\alpha(t)\,dG(\alpha)$$
$$\cdot\int_0^t e^{-sx}\int_0^\infty f_\alpha(x)\,dx\,dg(\alpha)$$
$$\geq\overline{F}(t)\int_0^t e^{-sx}f(x)\,dx.$$

Which complete the proof. □

2.3 Homogeneous Poisson Shock Model

An important application of ageing notion is shock models. Suppose that a device is subject to shocks occurring randomly in time according to a Poisson process with constant intensity λ. Suppose further that the device has probability $\overline{P}_k$ of surviving the first k shocks, where $1=\overline{P}_0\geq\overline{P}_1\geq\cdots$, then the survival function of the device is given by,

$$\overline{H}(t)=\sum_{k=0}^{\infty}\frac{(\lambda t)^k e^{-\lambda t}}{k!}\overline{P}_k,\ \ t\geq 0, \tag{2}$$
$$h(t)=\sum_{k=0}^{\infty}\frac{(\lambda t)^k e^{-\lambda t}}{k!}\lambda p_{k+1},\ \ t\geq 0, \tag{3}$$

where, $p_{k+1}=\overline{P}_k-\overline{P}_{k+1}$ is the probability mass function of N at point $k+1$.

This shock model has been studied by [5–10], and others.

Definition 2: A discrete distribution $P_k,\ k=0,1,\ldots,\infty$ with survival function $\overline{P}_k=1-P_k$ is said to have discreet $DLTTF$ if,

$$p_{r+1}\sum_{j=0}^{r}Z^j\overline{P}_j\geq\overline{P}_r\sum_{j=0}^{r}Z^j p_{j+1}. \tag{4}$$

Theorem 3: If P_k is discrete $DLTTF$, then $\overline{H}(t)$ given by (2) is $DLTTF$.

Proof: We need to show that

$$h(t)\int_0^t e^{-sx}\overline{H}(x)\,dx\geq\overline{H}(t)\,du\int_0^t e^{-sx}h(x)\,dx$$

using Eq. Eq. (2) and Eq. (3) we get, consider,

$$h(t)\int_0^t e^{-sx}\overline{H}(x)\,dx$$

$$=\left[\sum_{k=0}^{\infty}\frac{(\lambda t)^k e^{-\lambda t}}{k!}\lambda p_{k+1}\right]\int_0^t e^{-sx}\left[\sum_{m=0}^{\infty}\frac{(\lambda t)^m e^{-\lambda x}}{m!}\overline{P}_m\right]dx$$

$$=\left[\sum_{k=0}^{\infty}\frac{(\lambda t)^k e^{-\lambda t}}{k!}\lambda p_{k+1}\right]\sum_{m=0}^{\infty}\overline{P}_m\left[\int_0^t\frac{(\lambda t)^m}{m!}e^{-(\lambda+s)x}dx\right]$$

$$=\left(\frac{\lambda}{\lambda+s}\right)^{m+1}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{r=m+1}^{\infty}\frac{(\lambda t)^k e^{-\lambda t}}{k!}$$

$$\cdot\frac{[(\lambda+s)t]^r}{r!}e^{-(\lambda+s)t}\overline{P}_m p_{k+1}$$

$$=\left(\frac{\lambda}{\lambda+s}\right)^{m+1}\sum_{k=0}^{\infty}p_{k+1}\sum_{m=0}^{k}\frac{(\lambda t)^m e^{-\lambda t}}{m!}\overline{P}_m$$

$$\cdot\sum_{r=m+1}^{\infty}\frac{[(\lambda+s)t]^r}{r!}e^{-(\lambda+s)t},$$

by using the $DLTTF$ property

$$h(t)\int_0^t e^{-sx}\overline{H}(x)\,dx$$

$$\geq\left(\frac{\lambda}{\lambda+s}\right)^{m+1}\sum_{k=0}^{\infty}\overline{P}_k\sum_{m=0}^{k}\frac{(\lambda t)^m e^{-\lambda t}}{m!}p_{m+1}$$

$$\cdot\sum_{r=m+1}^{\infty}\frac{[(\lambda+s)t]^r}{r!}e^{-(\lambda+s)t}$$

$$\geq\left(\frac{\lambda}{\lambda+s}\right)^{m+1}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\frac{(\lambda t)^k e^{-\lambda t}}{k!}\overline{P}_k p_{m+1}$$

$$\cdot\sum_{r=m+1}^{\infty}\frac{[(\lambda+s)t]^r}{r!}e^{-(\lambda+s)t}$$

$$\geq\left[\sum_{k=0}^{\infty}\frac{(\lambda t)^k e^{-\lambda t}}{k!}\overline{P}_k\right]\int_0^t e^{-sx}\left[\sum_{m=0}^{\infty}\lambda p_{m+1}\frac{(\lambda t)^m e^{-\lambda x}}{m!}\right]dx$$

$$\geq\overline{H}(t)\,du\int_0^t e^{-sx}h(x)\,dx.$$

Which complete the proof. □

3. Testing exponentiality against $DLTTF$ class

Testing for classes of life distributions has been studied by many authors, see, [11–19] among other. In this section, a test statistic based on goodness of fit approach is presented for testing $H_0 : F$ is exponential against the alternative $H_1 : F$ belongs to $DLTTF$ class but not exponential. we use $\Lambda(s)$ as a measure of departure from exponentiality.

Lemma 1: Let X be a random variable with distribution F, then

$$\Lambda(s)=\int_0^{\infty}\left[\frac{1}{s}e^{-x}-\frac{1+s}{s}e^{-x(1+s)}\overline{F}(x)-\frac{1-s}{s}e^{-sx}\left(\int_x^{\infty}e^{-t}dF(t)\right)\right]dF(x)$$

Proof: Take the integral to Eq. (1) with respect to $F_0(t)$, then we have,

$$\begin{aligned}R.H.S&=\int_0^{\infty}e^{-t}\overline{F}(t)\int_0^t e^{-sx}f(x)\,dxdt\\&=\int_0^{\infty}e^{-sx}\int_x^{\infty}e^{-t}\overline{F}(t)\,dtdF(x)\\&=\int_0^{\infty}\left[e^{-x(1+s)}\overline{F}(x)-e^{-sx}\left(\int_x^{\infty}e^{-t}dF(t)\right)\right]dF(x).\end{aligned}\tag{5}$$

and,

$$\begin{aligned}L.H.S&=\int_0^{\infty}e^{-t}f(t)\int_0^t e^{-sx}\overline{F}(x)\,dxdt\\&=\int_0^{\infty}e^{-t}\left[\frac{1}{s}-\frac{1}{s}e^{-st}\overline{F}(t)-\frac{1}{s}\int_0^t e^{-sx}dF(x)\right]dF(t)\\&=\int_0^{\infty}\left[\frac{1}{s}e^{-x}-\frac{1}{s}e^{-x(1+s)}\overline{F}(x)-\frac{1}{s}e^{-sx}\left(\int_x^{\infty}e^{-t}dF(t)\right)\right]dF(x).\end{aligned}\tag{6}$$

Hence, from Eq. (5) and Eq. (6) the result follows. □

3.1 Empirical test statistic for $DLTTF$

To estimate $\Lambda(s)$, let $X_1, X_2, \ldots, X_n$ be a random sample from F. Let $\overline{F}_n(x)$ denote the empirical distribution of the survival function $\overline{F}(x)$ where,

$$\overline{F}_n(x)=\frac{1}{n}\sum_{i=1}^{n}I(X_i>x),\ dF_n(x)=\frac{1}{n}.$$

And let $\widehat{\Lambda}(s)$ be the empirical estimate of $\Lambda(s)$ where,

$$\widehat{\Lambda}(s)=\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\frac{1}{s}e^{-X_i}-\left[\frac{1+s}{s}e^{-(1+s)X_i}+\frac{1-s}{s}e^{-sX_i}e^{-X_j}\right]I(X_j>X_i)\right].\tag{7}$$

Set,

$$\phi(X_1,X_2)=\left[\frac{1}{s}e^{-X_1}-\left[\frac{1+s}{s}e^{-(1+s)X_1}+\frac{1-s}{s}e^{-sX_1}e^{-X_2}\right]I(X_2>X_1)\right].\tag{8}$$

The following theorem summarizes the asymptotic normality of $\widehat{\Lambda}(s)$.

Theorem 4: As $n\to\infty$, $\sqrt{n}\left(\widehat{\Lambda}(s)-\Lambda(s)\right)$ is asymptotically normal with mean 0 and variance σ^2 given as

$$\sigma^2(s)=Var\left[\frac{2}{1+s}e^{-X}-\frac{(s+3)^2}{2(1+s)(2+s)}e^{-(2+s)X}-\frac{1}{2(2+s)}\right],$$

under H_0 the variance tends to

$$\sigma_0^2(s)=\frac{8+3s}{12(4+s)(5+2s)}.\tag{9}$$

Proof: From Eq. (8) we set,

$$\phi(X_1) = E[\phi(X_1,X_2)|X_1] + E[\phi(X_2,X_1)|X_1]$$
$$= \frac{1}{s}e^{-X_1} - \int_{X_1}^{\infty}\left[\frac{1+s}{s}e^{-(1+s)x_1} + \frac{1-s}{s}e^{-sx_1}e^{-x_2}\right]e^{-x_2}dx_2 + \frac{1}{s}\int_0^{\infty}e^{-2x_2}dx_2$$
$$- \int_0^{X_1}\left[\frac{1+s}{s}e^{-(1+s)x_2} + \frac{1-s}{s}e^{-sx_2}e^{-x_1}\right]e^{-x_2}dx_2$$
$$= \frac{2}{1+s}e^{-X_1} - \frac{(s+3)^2}{2(1+s)(2+s)}e^{-(2+s)X_1} - \frac{1}{2(2+s)}$$

Hence

$$\sigma^2(s) = Var[\phi(X_1)].$$

Under H_0 the variance reduces to Eq. (9). □

4. The Pitman Asymptotic Relative Efficiency

In order to asses how good our proposed family of tests relative to others in the literature, we employ the concept of "Pitman's asymptotic relative efficiency"($PARE$) of proposed test. To present this, we evaluate the "Pitman's asymptotic efficiency"(PAE) where,

$$PAE(\Lambda(s)) = \frac{1}{\sigma_0}\left|\frac{d}{d\theta}\Lambda(s)\right|_{\theta\to\theta_0}.$$

For some commonly used distributions in reliability,

(i) Linear failure rate family,

$$\overline{F}_1(x) = \exp(-x-\theta x^2/2),\ x \ge 0, \theta \ge 0,$$

(ii) Makeham family,

$$\overline{F}_2(x) = \exp(-x-\theta(x+e^{-x}-1)), x \ge 0, \theta \ge 0,$$

(iii) Weibull family,

$$\overline{F}_3(x) = \exp(-x^{\theta}), x \ge 0, \theta \ge 1,$$

(iv) Gamma family,

$$\overline{F}_4(x) = \int_x^{\infty}e^{-u}u^{\theta-1}du/\Gamma(\theta),\ x > 0, \theta \ge 0.$$

Note that the exponential distribution is attained at $\theta_0 = 0$ in (i), (ii) , and $\theta_0 = 1$ in (iii), (iv)

Since

$$\Lambda_\theta(s) = \int_0^{\infty}\left[\frac{1}{s}e^{-x} - \frac{1+s}{s}e^{-x(1+s)}\overline{F}_\theta(x) - \frac{1-s}{s}e^{-sx}\left(\int_x^{\infty}e^{-t}dF_\theta(t)\right)\right]dF_\theta(x)$$

The $PAE(\Lambda_\theta(s))$ can be written as,

$$PAE(\Lambda_\theta(s)) = \frac{1}{\sigma_0}\left|\int_0^{\infty}\left[\frac{1}{s}e^{-x} - \frac{1+s}{s}e^{-x(1+s)}\overline{F}_\theta(x) - \frac{1-s}{s}e^{-sx}\left(\int_x^{\infty}e^{-t}dF_\theta(t)\right)\right]dF'_\theta(x) + \int_0^{\infty}\left[-\frac{1+s}{s}e^{-x(1+s)}\overline{F}'_\theta(x) - \frac{1-s}{s}e^{-sx}\left(\int_x^{\infty}e^{-t}dF'_\theta(t)\right)\right]dF_\theta(x)\right|_{\theta\to\theta_0}.$$

In this case, we obtain,

$$PAE\left(\Lambda_\theta(s), \overline{F}_1(x)\right) = \frac{1}{\sigma_0}\left|\frac{1}{12+4s}\right|,$$
$$PAE\left(\Lambda_\theta(s), \overline{F}_2(x)\right) = \frac{1}{\sigma_0}\left|\frac{1}{24+6s}\right|,$$
$$PAE\left(\Lambda_\theta(s), \overline{F}_3(x)\right) = \frac{1}{\sigma_0}\left|-\frac{s\log(4)+\log(64)-2(3+s)\log(3+s)}{4(1+s)(3+s)}\right|,$$
$$PAE\left(\Lambda_\theta(s), \overline{F}_4(x)\right) = \frac{1}{\sigma_0}\left|-\frac{s\log(4)+\log(16)-(3+s)\log(3+s)}{2(1+s)(2+s)}\right|.$$

Table 1 gives the efficiencies of our test $\Lambda_\theta(s)$ comparing with the tests given by [2, 16, 19] respectively.We have maximum value at $s = 0.05$.

Table 1. Comparison between the PAE of our test and some other tests

Distribution	$\Lambda_\theta(s)$	$\widehat{\Delta}_{rp}(\theta)$	$\delta^{(2)}_{F_n}$	$\Delta_j(\theta)$
LFR	0.45203	0.91287	0.217	0.4821
Makeham	0.22694	0.22823	0.144	2.4013
Weibull	1.10818	0.78785	0.05	-
Gamma	0.71644	0.34142	-	-

Also, Pittman asymptotic relative efficiency $(PARE)$ of our test is calculated where $PARE(T_1,T_2) = \frac{PAF(T_1)}{PAF(T_2)}$

5. Monte Carlo null distribution critical points

In practice, simulated percentiles for small samples are commonly used by applied statisticians and reliability analyst. We have simulated the upper percentile values for $90\%, 95\%, 98\%$ and 99%. Table 3, 4 contain the percentile values of the statistics $\widehat{\Lambda}(s)$ and the calculations are based on 10000 simulated samples of sizes $n = 5(5)50, 39$.

In view of Tables 3, 4, it is noticed that the critical values are increasing as the confidence level increasing and decreasing as the sample size increasing.

5.1 The Power Estimates

In this subsection, we present the power estimates of the test statistic $\widehat{\Lambda}(s)$ at the significance levels $\alpha = 0.05$ and $\alpha = 0.01$ respectively. These powers are estimated for LFR

Table 2. The asymptotic relative efficiencies for our test

Distribution	$PAE\left(\Lambda_\theta(s), \widehat{\Delta}_{rp}(\theta)\right)$	$PAE\left(\Lambda_\theta(s), \delta^{(2)}_{F_n}\right)$	$PAE\left(\Lambda_\theta(s), \Delta_j(\theta)\right)$
LFR	0.495174	2.083087	0.937627
Makeham	1.980589	1.575972	0.094507
Weibull	1.406587	22.16360	-
Gamma	2.098412	-	-

Table 3. Percentiles of $\widehat{\Lambda}(s)$ at $s = 0.05$.

n	90%	95%	98%	99%
5	2.64876	2.83321	3.04250	3.16480
10	1.22376	1.28831	1.36333	1.40197
15	0.78547	0.82257	0.86149	0.88702
20	0.58092	0.60395	0.63323	0.65138
25	0.46039	0.47854	0.49849	0.51047
30	0.38213	0.39641	0.41233	0.42399
35	0.32703	0.33988	0.35285	0.36234
39	0.29342	0.30482	0.31691	0.32542
40	0.28745	0.29827	0.31059	0.31764
45	0.25647	0.26595	0.27761	0.28495
50	0.23127	0.23979	0.25084	0.25905

Table 4. Percentiles of $\widehat{\Lambda}(s)$ at $s = 2.5$

n	90%	95%	98%	99%
5	0.10398	0.11731	0.13149	0.14053
10	0.07353	0.08634	0.09855	0.10661
15	0.05824	0.06958	0.08212	0.09028
20	0.04839	0.05909	0.06954	0.07808
25	0.04323	0.05260	0.06265	0.06979
30	0.03856	0.04707	0.05679	0.06311
35	0.03546	0.04304	0.05236	0.05823
39	0.03368	0.04144	0.04993	0.05499
40	0.03382	0.04173	0.04969	0.05451
45	0.03074	0.03777	0.04634	0.05214
50	0.02894	0.03607	0.04338	0.04755

and Weibull distributions. The estimates are based on 10000 simulated samples for sizes $n = 10, 20$ and 30 with parameter $\theta = 2, 3$ and 4.

Table 5. Power estimates at $s = 0.05$.

Distribution	θ	$n = 10$	$n = 20$	$n = 30$
Weibull	2	0.9994	1.0000	1.0000
	3	1.0000	1.0000	1.0000
	4	1.0000	1.0000	1.0000
LFR	2	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000
	4	1.0000	1.0000	1.0000

From Table 5, 6 it is noted that the power of the test increases by increases the value of the parameter θ and sample size n.

6. Applications

In this section, we apply our test to some real data-sets in the case of non censored data at 95% confidence level.

Table 6. Power estimates at $s = 2.5$.

Distribution	θ	$n = 10$	$n = 20$	$n = 30$
Weibull	2	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000
	4	1.0000	1.0000	1.0000
LFR	2	0.999	0.999	0.9996
	3	0.9997	0.9998	1.0000
	4	1.0000	1.0000	1.0000

Data-set #1.

Consider the data given in [20], these data represent 39 liver cancers patients taken from Elminia cancer center Ministry of Health - Egypt. In this case, we get $\widehat{\Lambda}(s) = 1.45394$ at $s = 0.05$ and $\widehat{\Lambda}(s) = 0.0595115$ at $s = 2.5$ and these value greater than the tabulated critical value in Tables 3, 4. This means that the set of data have $DLTTF$ property.

Data-set #2.

Consider the well-known Darwin data Fisher (1966) that represent the differences in heights between cross- and self-fertilized plants of the same pair grown together in one pot. It is easily to show that $\widehat{\Lambda}(s) = 0.206811$ at $s = 0.05$ and $\widehat{\Lambda}(s) = 0.0324909$ at $s = 2.5$, which is less than the critical value in Tables 3, 4. Then we accept H_0 which states that the data set have exponential property.

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