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Transmission Dynamic Response of Cholera Epidemic Model to Indirect and Direct Infectious Contact: Stability and Homotopy Analysis Method

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***Abstract*:** Cholera is generally a disease of the poor affecting regions without an enlightened sense of hygiene and access to safe drinking water. In this paper, a mathematical model for cholera transmission dynamics is formulated. The cholera model formulated is designed into compartments which lead to the system of nonlinear ordinary differential equations. The model is built on the assumption that cholera is contacted indirectly through the ingestion of *vibrio cholerae* in contaminated water and direct acquisition with infected individual through effective infectious contact. The model is analyzed in a positively invariant region to show that it is bounded, realistic in an epidemiological sense and mathematically well posed. The basic reproduction number R_{ch} is obtained and shown that if $R_{ch} < 1$, then cholera epidemic model at the cholera-free equilibrium solutions is locally and globally asymptotically stable. The numerical solutions of the model is obtained using the Homotopy Analysis Method (HAM) and graphical illustrations were presented.

***Keywords*:** cholera epidemic, basic reproduction number, local stability, global stability, homotopy analysis method.

1. Introduction

Cholera is a contagious disease which spread directly and indirectly through reservoir to human and human to human infectious contact. It is a gastrointestinal infection associated with clinical manifestations of severe vomiting, diarrhea, loss of body fluid and electrolytes and death. World Health Organization (WHO) fact sheet (2018) [1, 2], says between 1.3 to 4 million cases of cholera occurred, and between 21,000 to 143,000 people died of cholera epidemic in the world. Cholera poses a serious threat with its dangerous impact most felt in the poor third world countries [3–9]. Unsafe areas where cholera epidemic thrives are slums, refugee camps, dung hills and riversides. Proper medical facilities like safe toilet, clean water, environmental sanitation equipments are needed to be made available to forestall healthy living in these poor regions affected by cholera epidemic, in order to eliminate the disease in the human host environment. Literature of [10–17] have proved useful in gaining understanding in the complexities involved in cholera epidemics with the help of mathematical technique and analysis. The basic reproduction number is an important threshold in studying the stabilities of epidemic models [18]. It is defined as the expected number of secondary cases of cholera disease arising when a cholera infected individual is introduced into the population of individuals susceptible to cholera disease. [19–21], Proposed a model describing the concentration of *vibrio cholerae* in water and incorporating it into SIR model. The Homotopy Analysis Method (HAM) is a semi-analytical technique used in solving nonlinear, partial and ordinary differential equations [22, 23]. It helps in generating a convergent series solution for a nonlinear system. It was first proposed by Liao [24], introducing a convergence control parameter C_o to construct a homotopy on a general form of a differential system. In this work, a mathematical model describing the cholera dynamics incorporating different state variables and parameters is presented. HAM was employed to solve the model in this work. Also, the model is analyzed in a feasible region and shown to be locally and globally asymptotically stable at the cholera-free equilibrium solutions. The paper is organized as follows; Section 2 involves the model formulation and positivity analysis. Also the basic reproduction number is obtained. Section 3 discusses the local and global asymptotic stability analysis investigated at the cholera-free equilibrium solutions. Section 4 deals with the numerical result of the model using the HAM. Section 5 involves the graphical illustrations, using valid parameter values. Also, conclusion and recommendations were made.

2. Main Results

In this section, a five dimensional deterministic system of first order differential equation is considered. The human host population is subdivided into compartmental classes of state variables with parameters. The total human host population at time t, denoted by $N(t)$, is subdivided into five compartments of susceptible individuals (people who are at the risk of becoming but are not yet infected with cholera) denoted by $S(t)$, exposed individual (people who have been infected but are not yet infectious) denoted by $E(t)$, infected individuals (people who manifest cholera symptoms and are infectious) denoted by $I(t)$. The individuals who recovered from the cholera disease is denoted by $R(t)$, and the amount of *vibrio cholerae* in the environment is denoted by $B(t)$; Therefore, $N(t) = S(t) + E(t) + I(t) + R(t) + B(t)$.

In this model build-up, A is the influx rate of susceptible individuals into the host population. There is an effective infectious indirect contact rate between infected environmental sources and susceptible individual at the rate β_1, also the

direct human effective infectious contact rate is denoted β_2. The natural mortality rate associated with all sub-populations is denoted μ, while ω denote the loss of immunity to cholera infections after treatment. The progression rate from exposed to infected and infected to recovery sub-population is respectively denoted by γ and ρ, while cholera induced death rate is denoted by δ. The concentration of vibrios in environmental sources is denoted ξ and the natural death of vibrios is denoted δ_1 and K is the amount of vibrios present in the infected environment. Going by the assumptions, variables and parameter descriptions involved in the model formulation, the cholera deterministic model is given by

$$\begin{aligned}
\dot{S} &= A - \frac{\beta_1 SB}{(K+B)} + \beta_2 SI - \mu S + \omega R,\\
\dot{E} &= \frac{\beta_1 SB}{(K+B)} + \beta_2 SI - (\gamma+\mu)E,\\
\dot{I} &= \gamma E - (\rho+\delta+\mu)I,\\
\dot{R} &= \rho I - \mu R - \omega R,\\
\dot{B} &= \xi I - \delta_1 B.
\end{aligned} \tag{1}$$

Subject to the initial conditions $S(0) = S_o$, $E(0) = E_o$, $I(0) = I_o$, $R(0) = R_o$, $B(0) = B_o$.

2.1 Positivity and Boundedness

It is assumed that the initial condition of the model is non-negative and it is necessary to show that the solutions of the model are positive.

Theorem 1: Let

$$\omega = \{(S,E,I,R,B) \in \Re^{+5} : S_o > 0, E_o > 0, I_o > 0, R_o > 0, B_o > 0\},$$

then the solutions of S, E, I, R, B are positive for $t \geq 0$.

Proof: From the model system Eq. (1), the addition of the total state equations yield,

$$N = A - \mu N + I\xi - Id - \delta B. \tag{2}$$

In the absence of cholera induced mortality and concentration of vibrios,

$$\dot{N} = A - \mu N(t). \tag{3}$$

on solving Eq. (3), one obtains

$$\lim_{t\to\infty} N \leq \frac{A}{\mu} - \left[\frac{(A-\mu N)}{\mu}\right] e^{-\mu t} = \frac{A}{\mu}. \tag{4}$$

Hence, all the solutions of model Eq. (1) are positive. The set $\omega = \left[\left\{(S,E,I,R,B) \in \Re^{+5} | S+E+I+R+B \leq \frac{A}{\mu}\right\}\right]$ is positively invariant and well posed. Hence, the proof. □

2.2 Equilibrium Solutions

In order to analyze the asymptotic stability of the model system Eq. (1) at the local and global domain, model system Eq. (1) is made static i.e., by obtaining the time-independent solutions. The two nonnegative equilibrium solutions which are the cholera-free and cholera-present solution is obtained. When $I = B = 0$, the cholera-free equilibrium solutions are given by

$$S, E, I, R, B = \left(\frac{A}{\mu}, 0, 0, 0, 0\right). \tag{5}$$

Also, when $E = I = B \neq 0$, the cholera-present equilibrium solutions are given by,

$$\begin{aligned}
S^{**} &= \frac{(A+\omega R)}{\beta_1 K + B^{**} + \beta_1 I + \mu},\\
E^{**} &= \frac{\beta_1 \frac{B^{**}}{(K+S)} + \beta_2 I^{**})S^{**}}{\gamma+\mu},\\
I^{**} &= \frac{\gamma E^{**}}{\rho+\delta+\mu},\\
R^{**} &= \frac{\rho E^{**}}{\mu+\omega},\\
B^{**} &= \frac{\xi I^{**}}{\delta_1}.
\end{aligned} \tag{6}$$

2.3 Basic Reproduction Number R_{ch}

The threshold that governs the spread of a disease, called the basic reproduction number is obtained [18–20].

Theorem 2: Define $X_s = \{x = 0 | x_i, i = 1,2,3,\ldots\}$ in order to obtain R_{ch}, we distinguish new cholera infections from other changes in the populations. Let $F_i(x)$ be the rate of new manifestation of cholera disease symptoms in compartment i and V_{i+} be the rate at which individuals move into compartment i through other means and V_{i-} be the rate at which individual move out of compartment i. Then, $x_i = f_i(x) = F_i(x) - V_i(x)$, and $V_i(x) = V_{i-} - V_{i+}$, where $i = 1,2,3,\ldots,n$ F is a non-negative matrix and V is a non-singular matrix.

Proof: We apply the next generation matrix method [18, 19], to the model equations starting with newly infective classes so that

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \frac{\beta_2 A}{\mu} & 0 & 0\\ 0 & \gamma & 0 & 0 & 0\\ 0 & 0 & \rho & 0 & 0\\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{7}$$

$$V = \begin{pmatrix} -\mu & 0 & \frac{\beta_2 A}{\mu} & \omega & 0\\ 0 & -(\gamma+\mu) & \frac{\beta_2 A}{\mu} & 0 & 0\\ 0 & \gamma & -(\rho+\delta+\mu) & 0 & 0\\ 0 & 0 & \rho & -(\omega+\mu) & 0\\ 0 & 0 & \xi & 0 & -\delta_1 \end{pmatrix} \tag{8}$$

and

$$V^{-1} = \begin{pmatrix} \frac{1}{-\mu} & \frac{-\omega\rho\gamma}{(\rho+\delta+\mu)(\rho+\mu)(\mu+\omega)\mu} & \frac{-\omega\rho}{(\rho+\delta+\mu)(\mu+\omega)\mu} & \frac{\omega\mu}{\omega+\mu}\mu & 0\\ 0 & -\frac{1}{(\gamma+\mu)} & 0 & 0 & 0\\ 0 & \frac{\gamma}{(\rho+\mu)(\rho+\delta+\mu)} & \frac{1}{(\rho+\delta+\mu)} & 0 & 0\\ 0 & 0 & \rho & -(\omega+\mu) & 0\\ 0 & 0 & \xi & 0 & -\delta_1 \end{pmatrix}. \tag{9}$$

Therefore, the largest eigenvalue is the basic reproduction number R_{ch} of cholera model considered is given by

$$R_{ch}(FV^{-1}) = \frac{(A(\beta_1 M + \beta_2 I)\omega\rho)}{((\rho+\delta+\mu)(\gamma+\mu)(\mu+\omega)\mu)}. \quad \square$$

3. Stability Analysis

3.1 Local Stability Analysis of Cholera-Free Equilibrium

Theorem 3: The Cholera-free equilibrium solutions are locally asymptotically stable if $R_{ch} < 1$ and unstable when $R_{ch} > 1$.

Proof: The Jacobian matrix of Eq. (1) at the cholera-free equilibrium solutions is given by

$$\begin{pmatrix} -Z_1-\mu & 0 & -\beta_1 S & \omega & 0 \\ 0 & -Z_2-\lambda & -\beta_2 S & 0 & 0 \\ 0 & \gamma & -Z_3-\lambda & 0 & 0 \\ 0 & 0 & \rho & -(\mu-\omega) & 0 \\ 0 & 0 & -\xi & 0 & -\delta_1-\lambda \end{pmatrix} \tag{10}$$

so that,

$$(Z_1-\lambda)(Z_2-\lambda)(Z_3-\lambda)[(\mu-\omega)(\delta-\lambda)] = 0. \tag{11}$$

It is however observed that

$$\begin{aligned} \lambda_1 &= Z_1 = -\mu, \\ \lambda_2 &= Z_2 = -(\gamma+\mu), \\ \lambda_3 &= Z_3 = -(\rho+\delta+\mu) \end{aligned} \tag{12}$$

and

$$[(\mu-\omega)-\lambda)(\delta-\lambda)] = 0. \tag{13}$$

Let $L_1 = (\mu-\omega)$and $L_2 = \delta$ so that,

$$\begin{aligned} &(L_1-\lambda)(L_2-\lambda) = 0, \\ &L_1L_2 - \lambda L_2 + \lambda^2 = 0, \\ &\lambda^2 - \lambda(L_1+L_2) + L_1L_2 = 0. \end{aligned} \tag{14}$$

By the principle of Routh Hurwitz [19], Eq. Eq. (13) has strictly negative real root if and only if $L_1 > 0, L_2 > 0$ and $L_1 > L_2$. Obviously, we see that L_1 is positive because it is the sum of positive variable, for L_2 to be positive, $1 - R_{ch}$ must be positive which leads to $R_{ch} < 1$. Therefore the cholera-free equilibrium will be locally asymptotically stable if and only if $R_{ch} < 1$. $\square$

3.2 Global Stability of the Cholera-Free Equilibrium

Theorem 4: The cholera-free equilibrium solutions are globally asymptotically if $R_{ch} < 1$.

Proof: The comparison theorem is applied by considering the state equation with infected compartments only in Eq. (1). That is,

$$\begin{aligned} \dot{E} &= \left(\frac{\beta_1 B}{K+B} + \beta_2 I\right) S - (\gamma+\mu)E \\ \dot{I} &= \gamma E - (\rho+\delta+\mu)I \\ \dot{B} &= \xi I - \delta_1 B. \end{aligned} \tag{15}$$

Eq. (15) can be re-written as

$$\begin{pmatrix} \dot{E} \\ \dot{I} \\ \dot{B} \end{pmatrix} = F \begin{pmatrix} E \\ I \\ B \end{pmatrix} - V \begin{pmatrix} E \\ I \\ B \end{pmatrix} \leq (F-V) \begin{pmatrix} E \\ I \\ B \end{pmatrix}. \tag{16}$$

Where F and V had been defined earlier in theorem 2.

Since F and V have negative real part when $R_{ch} < 1$, the linearized differential inequality of Eq. (36) is stable whenever $R_{ch} \leq 1$. By the comparison theorem,

$$(E, I, B) \to (0,0,0) \;\; t \to \infty. \tag{17}$$

Substituting $E = I = B = 0$ in Eq. (1) yields

$$(S, R) = \left(\frac{A}{\mu}, 0\right) \;\; t \to \infty. \tag{18}$$

Therefore, the cholera- free equilibrium is globally asymptotically stable whenever $R_{ch} < 1$. $\square$

4. Basic Idea of Homotopy Analysis Method (HAM)

Consider $N[U(t) = 0]$, where N is any operator, $U(t)$ is unknown function, 't' is the independent variable, $U_o(t)$ is the intial guess of the exact solution, $U(t)$, $h \neq 0$ is an auxiliary parameter, $H(t) \neq 0$ is auxiliary function, and L is an auxiliary linear operator with the property that $L[U(t) = 0]$ when $U(t) = 0$. Then, using $p \in [0,1]$ as an embedding parameter, we construct a homotopy, such that

$$(1-\rho)L[f(t;\,\rho) - U_o(t)] - \rho h H(t) N[f(t;\,\rho)] = \hat{H}. \tag{19}$$

We can choose the initial guess $U_o(t)$, the auxiliary linear operator L, the nonzero auxiliary parameter 'h' and the auxiliary function $H(t)$ suitably. Equating Eq. (19) to zero order deformation equation, yield

$$(1-\rho)L[f(t;\,\rho) - U_o(t)] = \rho h H(t) N[f(t;\,\rho)]. \tag{20}$$

When $\rho = 0$, the zero order deformation become

$$L[f(t;\,\rho) - U_o(t)] = 0. \tag{21}$$

$f(t;\,0) - U_o(t)$ is the initial approximation when $\rho = 1$, since $h \neq 0$ and $H(t) \neq 0$, the zero-order deformation of Eq. (20) is equivalent to $N[f(t;1)] = 0$. Therefore, $f(t;1) - U(t)$, is the exact solution.

As the embedding parameter p increase from 0 to 1, $f(t;p)$ varies continuously from the initial approximation $U_o(t)$ to the exact solution $U(t)$. Such kind of continuous variation is called deformation in homotopy. $f(t;p)$ can be expanded using Taylor's Theorem such that

$$f(t;p) = U_o(t) + \sum m = 1 U_m(t) p^m \tag{22}$$

where

$$U_m(t) = \frac{1}{m!} \frac{\partial^m f(t;p)}{\partial p^m}\Big|_{p=0}. \tag{23}$$

If the initial guess $U_o(t)$,the auxiliary linear operator L, the non-zero auxiliary parameter h and the auxiliary function $H(t)$ are properly chosen, then

- The solution $f(t;p)$ of the zero order deformation equation exists for all $\rho \in [0,1]$.
- $\frac{\partial^m f(t;p)}{\partial^m}\Big|_{(p=0)}$ exists for $m = 1,2,3,\ldots,m$.
- The power series of $f(t;p)$ converges at $p = 1$.

Under these assumptions the solution series, reduces to

$$f(t;\ p) = U_o(t) + \sum m = 1 U_m(t) \tag{24}$$

We define the vector $U_n(t) = \{U_o(t), U_1(t), U_2(t), \ldots, U_n(t)\}$.

According to definition, $U_m(t)$ can be derived from the zero-order deformation. Differentiating the zero-order deformation equation m times with respect to p and then dividing by $m!$ and finally setting $p = 0$, we have the so called $m^t h$ order deformation equation

$$L[U_m(t) - \alpha_{m-1}(t)] = hH(t) R_m U_{m-1}(t) \tag{25}$$

Where

$$R_m(U_{m-1}(t)) = \frac{1}{(m-1)!} \frac{(\partial^{m-1} N[f(t;p)])}{\partial_p{}^{m-1}}, \tag{26}$$

and $\alpha_m = 0$ when $m \leq 1$ and $\alpha_m = 1$ when $m = 1$. Consider the system,

$$\begin{aligned}
\dot{S} &= A - \beta_1 SM - \beta_2 SI - \mu S + \omega R,\\
\dot{E} &= \beta_1 SM + \beta_2 SI - (\gamma + \mu) E,\\
\dot{I} &= \gamma E - (\rho + \delta + \mu) I,\\
\dot{R} &= \rho I - \mu R - \omega R,\\
\dot{B} &= \delta B - \xi I,
\end{aligned} \tag{27}$$

subject to the initial conditions $S(0) \geq 0$, $E(0) \geq 0$, $I(0) \geq 0$, $R(0) \geq 0$, $B(0) \geq 0$, where $M = \frac{B}{(K+B)}$. Then

$$\begin{aligned}
LS(t) - LS^o(t) =& z[A - \beta_1 SM - \beta_2 SI - \mu S + \omega R\\
& - LS^o(t)],\\
LE(t) - LE^o(t) =& z[\beta_1 SM + \beta_2 SI - (\gamma + \mu) E\\
& - LE^o(t)],\\
LI(t) - LI^o(t) =& z[\gamma E - (\rho + \delta + \mu) LI^o(t)],\\
LR(t) - LR^o(t) =& z[\rho - \mu R - \omega R - LR^o(t)],\\
LB(t) - LB^o(t) =& z[\delta B - \xi I - LB^o(t)].
\end{aligned} \tag{28}$$

In the following we assume the solution for system Eq. (28) above in the form

$$\begin{aligned}
S^*(t) &= S_o^*(t) + zS_1^*(t) + z^2 S_2^*(t) + \cdots,\\
E^*(t) &= E_o^*(t) + zE_1^*(t) + z^2 E_2^*(t) + \cdots,\\
I^*(t) &= I_o^*(t) + zI_1^*(t) + z^2 I_2^*(t) + \cdots,\\
R^*(t) &= R_o^*(t) + zR_1^*(t) + z^2 R_2^*(t) + \cdots,\\
B^*(t) &= B_o^*(t) + zB_1^*(t) + z^2 B_2^*(t) + \cdots.
\end{aligned} \tag{29}$$

Using equation Eq. (28) and Eq. (29) and comparing the coefficient of the same power, we set;

$$\begin{aligned}
&LS(t) - Ls^o(t) = 0,\\
&LE(t) - LE^o(t) = 0,\\
&LI(t) - LI^o(t) = 0,\\
&LR(t) = LR^o(t) = 0,\\
&LB(t) - LB^o(t) = 0,
\end{aligned} \tag{30}$$

and

$$\begin{aligned}
LS_1^*(t) &= [(A - \beta_1 S_o^* M - \beta_2 S_o^* I_o^* - \mu S_o^* + \omega R_o^* - LS^o(t),\\
LE_1^*(t) &= [\beta_1 S_o^* M + \beta_2 S_o^* I_o^* - (\gamma + \mu) E_o^*(t)],\\
LI_1^*(t) &= [\gamma E_o^* - (\rho + \delta + \mu) I_o^* - LI_o^*(t)],\\
LR_1^*(t) &= [\gamma I_o^* - \mu R_o^* - \omega R_o^* - LR_o^*(t)],\\
LB_1^*(t) &= \delta B_o^* - I_O^*.
\end{aligned} \tag{31}$$

With the condition;

$$S_1^*(t) = 0, E_1^*(t) = 0, I_1^*(t) = 0, R_1^*(t) = 0, B_1^*(t) = 0. \tag{32}$$

Also,

$$\begin{aligned}
LS_2^*(t) &= A - \beta_1 S_1^* M + \beta_2 (S_o^* I_1^* + S_1^* I_o^*) - S_1^* + \omega R_1^*,\\
LE_2^*(t) &= \beta_1 S_1^* M + \beta_2 (S_o^* I_1^* + S_1^* I_o^*) - (\gamma + \mu) E_1^*,\\
LI_2^*(t) &= \gamma E_1^* - (\rho + \delta + \mu) I_2^*,\\
LR_2^*(t) &= \rho I_1^* - \mu R_1^* - \omega R_1^*,\\
LB_2^*(t) &= \delta B_1^* - I_1^*.
\end{aligned} \tag{33}$$

With the condition;

$$S_2^*(t) = 0, E_2^*(t) = 0, I_2^*(t) = 0, R_2^*(t) = 0, B_2^*(t) = 0.$$

Also,

$$\begin{aligned}
LS_3^*(t) =& A - \beta_1 S_2^* M - \beta_2 (S_o^* I_2^* + 2S_1^* I_o^* + S_2^* I_o^*)\\
& - \mu S_2^* + \omega R_2^*,\\
LE_3^*(t) =& \beta_1 S_1^* M - \beta_2 (S_1^* I_2^* + 2S_1^* I_1^* + S_2^* I_o^*)\\
& - (\gamma + \mu) E_2^*,\\
LI_3^*(t) =& \gamma E_2^* - (\rho + \delta + \mu) I_2^*,\\
LR_3^*(t) =& \rho I_2^* - \mu R_2^* - \omega R_2^*,\\
LB_3^*(t) =& \delta_1 B - I_2^*.
\end{aligned} \tag{34}$$

With the conditions;

$$S_3^*(t)=0,\ E_3^*(t)=0,\ I_3^*(t)=0,$$
$$R_3^*(t)=0,\ B_3^*(t)=0.$$

To find the solution, we take $z=1$ in the system Eq. (29), to obtain;

$$\begin{aligned}
S^*(t)&=S_o^*(t)+S_1^*(t)+S_2^*(t)+\cdots,\\
E^*(t)&=E_o^*(t)+E_1^*(t)+E_2^*+\cdots,\\
I^*(t)&=I_o^*(t)+I_1^*(t)+I_2^*(t)+\cdots,\\
R^*(t)&=R_o^*(t)+R_1^*(t)+R_2^*(t)+\cdots,\\
B^*(t)&=B_o^*(t)+B_1^*(t)+B_2^*(t)+\cdots.
\end{aligned}\tag{35}$$

For the zero order problem or z^o,

$$S_o^*(t)=A_1,\ E_o^*(t)=A_2,\ I_o^*(t)=A_3,$$
$$R_o^*(t)=A_4,\ B_o^*(t)=A_5.$$

The first order problem or z^1 is given by

$$\begin{aligned}
S_1^*(t)&=A-\beta_1e_1M-\beta_2e_1e_3-\mu e_1+\omega e_4,\\
E_1^*(t)&=\beta_1e_1M+\beta_2e_1e_3-(\gamma+\mu)e_2,\\
I_1^*(t)&=\gamma e_1-(\rho+\delta+\mu)e_3,\\
R_1^*(t)&=\rho e_3-\mu e_1-\omega e_4,\\
B_1^*(t)&=\delta e_5-\xi e_3.
\end{aligned}\tag{36}$$

So that

$$S_o^*(t)=A_1=e_1,\ E_o^*(t)=A_2=e_2,\ I_o^*(t)=A_3=e_3,$$
$$R_o^*(t)=A_4e_4,\ B_o^*(t)=A_5e_5.$$

The second order problem or z^2 is given by

$$\begin{aligned}
S_2^*(t)=&-\beta_1\Big(A-\beta_1e_1m-\beta_2e_1e_3-\mu e_1-\omega e_4\Big)m\frac{t^2}{2}\\
&-\beta_2[e(\gamma E-(\rho+\delta+\mu)+e_3(A-\beta_1e_1m-\beta_2e_1e_3\\
&-\mu e_1+\omega e_4))m]\frac{t^2}{2}\\
&-[\mu(A-\beta_1e_1m-\beta_2e_1e_3-\mu e_1+\omega e_4)m]\frac{t^2}{2}\\
&+\omega(\rho E_3-\mu E_4-\omega E_4)\frac{t^2}{2}
\end{aligned}$$

$$\begin{aligned}
E_2^*(t)=&(\beta_1e_1m-\beta_2e_1e_3-\mu E_1+\omega e_4)\frac{t^2}{2}\\
&+\beta_2(e_1(\gamma e_1-(\rho+\delta+\mu)e_3+e_3\\
&(A-\beta_1e_1m-\beta_2e_1e_3-Me_1+\omega e_4)\frac{t^2}{2}\\
&-(\gamma+\mu)(\beta_1e_1m+\beta_2e_1e_3-(\gamma+\mu)e_2)\frac{t^2}{2},
\end{aligned}$$

$$\begin{aligned}
I_2^*(t)=&\gamma(\beta_1e_1m+\beta_2e_1e_3-(\gamma+\mu)e_2)\frac{t^2}{2}\\
&-(\rho+\delta+\mu)\frac{t^2}{2},
\end{aligned}$$

$$\begin{aligned}
R_2^*(t)=&\rho(\gamma e_1-(\rho+\delta+\mu)\frac{t^2}{2}\\
&-\mu(\rho e_3-\mu(\rho e_3-\mu e_4-\omega e_4)\frac{t^2}{2}\\
&-\omega(\rho e_3-\mu e_4-\omega e_4)\frac{t^2}{2},\\
B_2^*(t)=&(\delta B-(\gamma e_1-(\rho+(\gamma e_1-(\rho+\delta+\mu)))))\frac{t^2}{2}.
\end{aligned}\tag{37}$$

5. Numerical Simulations

Table 1. Parameter and Variable Descriptions

Parameters/Variables	Values	Sources
S	50	Assumed
E	30	Assumed
I	10	Assumed
R	20	Assumed
B	15	Assumed
β_1	0.121	[12, 14]
β_2	0.11	[12, 14]
A	10	[23, 24]
γ	0.14	[14, 23]
δ	0.0012	[14, 23]
ρ	0.31	[14, 17]
ξ	0.512	[14, 17]
μ	0.21	[14, 17]
ω	0.22	[17, 23]
κ	0.0014	[12, 14]

Table 2. Numerical results for first order problem z^1

Time(t)	S(t)	E(t)	I(t)	R(t)	B(t)
0	0	0	0	0	0
0.2	1.9956	0.0009936	-0.10420024	-0.00237	-0.00982
0.4	3.9912	0.019872	-0.20840048	-0.00474	-0.01964
0.6	5.9868	0.029808	-0.31260072	-0.00711	-0.02946
0.8	7.9824	0.039744	-0.41680096	-0.00948	-0.03928
1.0	9.978	0.049680	-0.5210012	-0.01185	-0.0491

Table 3. Numerical results for second order problem z^2

Time(t)	S (t)	E (t)	I (t)	R (t)	B (t)
0	0.00814	-0.00814	0	0	0
0.2	-0.0031824	0.0419392	-0.0104226	-0.0031283	0.005695
0.4	-0.151716	0.192177	-0.0416904	-0.0125131	0.02278
0.6	-0.351536	0.442573	-0.0938034	-0.0281545	0.051255
0.8	-0.631284	0.793127	-0.1667616	-0.050052	0.09112
1.0	-0.99096	1.24384	-0.260565	-0.078207	0.014238

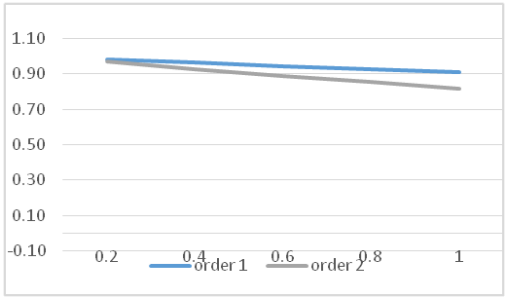

Figure 1. Graph of susceptible individuals $S(t)$ in the population at given time t

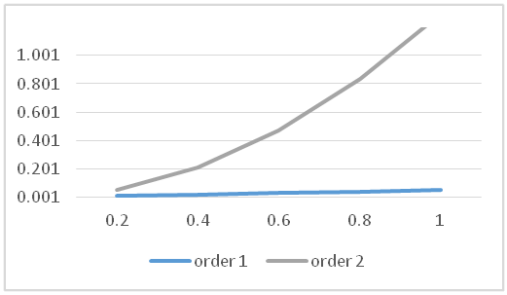

Figure 2. Graph of exposed individuals $E(t)$ in the population at given time t

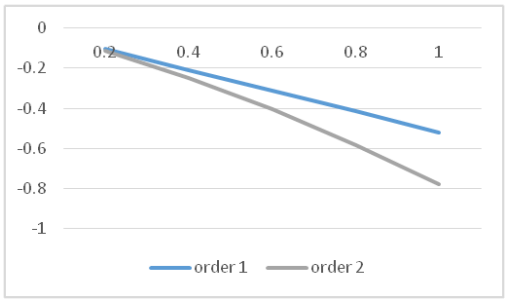

Figure 3. Graph of infected individuals $I(t)$ in the population at given time t

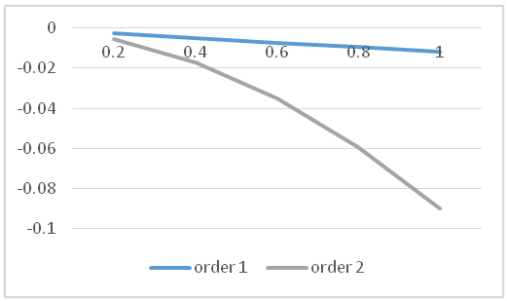

Figure 4. Graph of recovered individuals $R(t)$ in the population at given time t

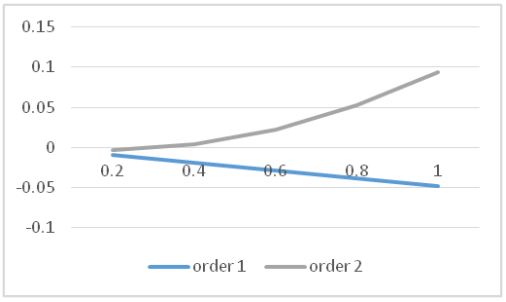

Figure 5. Graph of infected environmental sources $B(t)$ in the population at given time t

6. Conclusion and Recommendation

The model has two nonnegative equilibria, namely the cholera-free and cholera-present equilibrium and their existence and stability were analyzed. The analysis of the model also produces a threshold parameter, R_{ch} which is the basic reproduction number of cholera epidemic. It was noted that when $R_{ch} < 1$, the cholera-free equilibrium is symptomless and this indicates the cholera will not persist in the population. Also when $R_{ch} < 1$, the cholera-free equilibrium states exist and become globally stable. The equilibrium analysis show that the basic reproduction number of cholera for the model play an essential role in determining control measures against endemicity of cholera disease. The numerical simulation was carried out using HAM, and graphical illustration is presented to show the behavior of each sub-population towards cholera elimination. This work can be further extended by considering seasonality, age structure, spatiotemporal dynamics, optimal control and cost effective analysis.

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