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Further Characterization of $\omega-$Order Reversing Partial Contraction Mapping as a Compact Semigroup of Linear Operator

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Abstract: This paper consists of compact results on $\omega-$order reversing partial contraction mapping in semigroup of linear operator by giving special onsiderations to equicontinuous semigroups.

Keywords: ω-order-reserving partial contraction mapping (ω-$ORCP_n$), equicontinuous semigroups, C_0-semigroup, compact semigroup.

1. Introduction

Compactness in semigroup of linear operator can not be underestimated in the theory of semigroup of linear operators because of it importance in C_0-semigroup since it lays emphasis on its closeness, linear and equiontinuous nature. Suppose X is a Banach space, $X_n \subseteq X$ be a finite set, $(T(t))_{t\geq 0}$ the C_0-semigroup, $\omega - ORCP_n$ be ω-order-reversing partial contraction mapping which is an example of C_0-semigroup, $\omega - ORCP_n \subseteq ORCP_n$ (Order Reversing Partial Contraction Mapping). Let $Mm(\mathbb{N}\cup 0)$ be a matrix, $L(X)$ the bounded linear operator in X, P_n, the partial transformation semigroup, $\rho(A)$ a resolvent of A, where A is the generator of a semigroup of linear operator. This paper will focus on results of compactness on $\omega - ORCP_n$ in a semigroup of linear operator called C_0-semigroup.

Balakrishnan [1], proved fractional powers of closed operators and the semigroup generated by them. Banach [2], established and introduced the concept of Banach space. Engel and Nagel [3], established one-parameter semigroup for linear evolution equations. McBride [4], proved some semigroup of linear operators. Pazy [5], obtained some results on the differentiability and compactness of semigroup of linear operator. Rauf and Akinyele [6], obtained ω-order-preserving partial contraction mapping and established its properties, also in [7], Rauf *et.al.* established some results of stability and spectra properties on semigroup of linear operator. Vrabie [8], introduced some compactness criteria in $C(0,T;X)$ for subsets of solution of non-linear evolution equations governed by accretive operators. Vrabie [9], characterized new generators of linear compact semigroups and also deduced compactness method for nonlinear evolutionsin [10]. Furthermore, he obtained compactness in L^p of the set of solutions to linear evolution equation, qualitative problems for differential equations and control theory in [11]. In [12], Vrabie established some results of C_0-semigroup and its applications. Yosida [13], established some results on differentiability and representation of one-parameter semigroup of linear operators.

2. Preliminaries

Definition 1 (C_0-Semigroup [12]) A C_0-Semigroup is a strongly continuous one parameter semigroup of bounded linear operator on Banach space.

Definition 2 (ω-$ORCP_n$ [6]) A transformation $\alpha \in P_n$ is called ω-order-reversing partial contraction mapping if $\forall x,y \in$Dom$\alpha : x \leq y \implies \alpha x \geq \alpha y$ and at least one of its transformation must satisfy $\alpha y = y$ such that $T(t+s) = T(t)T(s)$ whenever $t,s > 0$ and otherwise for $T(0) = I$.

Definition 3 (Compact Semigroup [3]) A C_0-semigroup is compact if for each $t > 0$, $T(t)$ is a compact operator.

Definition 4 (Equicontinuous [12]) A C_0-semigroup $\{T(t); t \geq 0\}$ is equicontinuous if the function $t \to T(t)$ is continuous from $(0,+\infty)$ to $L(X)$ endowed with the uniform operator norm $\|.\|_{L(X)}$.

Example 1: 2×2 matrix $[M_m(\mathbb{N}\cup\{0\})]$.
Suppose

$$A = \begin{pmatrix} 2 & 2 \\ 2 & - \end{pmatrix}$$

and let $T(t) = e^{tA}$, then

$$e^{tA} = \begin{pmatrix} e^{2t} & e^{2t} \\ e^{2t} & I \end{pmatrix}.$$

Example 2: 3×3 matrix $[M_m(\mathbb{N}\cup\{0\})]$.
Suppose

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 3 & 2 & 2 \end{pmatrix}$$

and let $T(t) = e^{tA}$, then

$$e^{tA} = \begin{pmatrix} e^{3t} & e^{2t} & e^{t} \\ e^{2t} & e^{2t} & e^{t} \\ e^{3t} & e^{2t} & e^{2t} \end{pmatrix}.$$

Example 3: 3×3 matrix $[M_m(\mathbb{C})]$, we have for each $\lambda > 0$ such that $\lambda \in \rho(A)$ where $\rho(A)$ is a resolvent set on X. Suppose we have

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 3 & 2 & 2 \end{pmatrix}$$

and let $T(t) = e^{tA_\lambda}$, then

$$e^{tA_\lambda} = \begin{pmatrix} e^{3t\lambda} & e^{2t\lambda} & e^{t\lambda} \\ e^{2t\lambda} & e^{2t\lambda} & e^{t\lambda} \\ e^{3t\lambda} & e^{2t\lambda} & e^{2t\lambda} \end{pmatrix}.$$

Example 4: Take X to be one of the the sequence space ι^p, $1 < p < \infty$ or C_0. For every sequence $(\alpha_n)_{n\in\mathbb{N}} \subset \mathbb{R}_+$, the multiplication operator $A(x_n)_{n\in\mathbb{N}} = (i\alpha_n x_n)_{n\in\mathbb{N}}$ with maximal domain generates a semigroup of isometries on X; since each canonical basis vector is an eigenvector of A with eigenvalue $i\alpha_n$, it follows that the strong operator closure of multipliation semigroup $T(t)_{t\geq 0}$ with

$$T(t)(x_n)_{n\in\mathbb{N}} = (e^{i\alpha nt}x_n)_{n\in\mathbb{N}},\ t \geq 0,$$

is strongly compact semigroup.

Theorem 1: A linear operator $A : D(A) \subseteq X \to X$ is the infinitesimal generator for a C_0-semigroup of contraction if and only if

i. A is densely defined and closed,

ii. $(0,+\infty) \subseteq \rho(A)$ and for each $\lambda > 0$, we have

$$\|R(\lambda,A)\|_{L(X)} \leq \frac{1}{\lambda}.$$

3. Main Results

In this section, results of equicontinuous and compact semigroup on ω-$ORCP_n$ in semigroup of linear operator (C_0-semigroup) were considered:

Theorem 2: Suppose $A : D(A) \subseteq X \to X$ is the infinitesimal generator of a C_0-semigroup of contractions, $\{T(t); t \geq 0\}$ where $A \in \omega$-$ORCP_n$. Then $\{T(t); t \geq 0\}$ is equicontinuos if and only if, for each $\alpha \in (0,1)$, we have

$$\lim_{n\to\infty} (I - t/nA)^{-n} = T(t)$$

in the usual sup-norm topology of $C([\alpha, 1/\alpha]; L(X))$.

Proof: To prove the theorem, we need to assert that for each $a \in (0,1)$ and each $b \in (1,+\infty)$ we have

$$\lim_{n\to\infty} \frac{n^{n+1}}{n!}\int_0^a (ve^{-v})^n dv = 0 \tag{1}$$

and

$$\lim_{n\to\infty} \frac{n^{n+1}}{n!}\int_b^{+\infty} (ve^{-v})^n dv = 0. \tag{2}$$

We need to show that Eq. (1) and Eq. (2) converge accordingly. Since $t \to te^{-t}$ is nondecreasing on $[0,1]$, it follows that

$$\int_0^a (ve^{-v})^n dv \leq a(ae^{-a})^n.$$

On the other hand, $ve^{-v} < e^{-1}$ for each $v > 0$, $v \neq 1$, and accordingly

$$\lim_{n\to\infty} vn(ve^{-v}e)^n = 0 \tag{3}$$

for each $v > 0$, $v \neq 1$. Observing that, from Stirling's formula $\lim_{n\to\infty} \frac{n!}{\sqrt{2\Pi}n^{n+1/2}e^{-n}} = 1$, it follows that

$$\lim_{n\to\infty} \frac{n^n e^{-n}}{n!} = 0, \tag{4}$$

we btain

$$0 \leq \lim_{n\to\infty} \frac{n^{n+1}}{n!}\int_0^a (ve^{-v})^n dv$$
$$\leq \lim_{n\to\infty} an(ae^{-a}e)^n \frac{n^n e^{-n}}{n!} = 0$$

which proves Eq. (1) in the assertion. Let us observe that

$$\frac{n^{n+1}}{n!}\int_b^{+\infty} (ve^{-v})^n dv = e^{-nb}\sum_{k=0}^{n} \frac{(nb)^k}{k!} \tag{5}$$

for each $n \in \mathbb{N}^*$ and $b \geq 0$. Assume $b > 1$, we have

$$\frac{(nb)^k}{k!} \leq \frac{(nb)^n}{n!}$$

for $k = 1, 2, \ldots, n-1$, and accordingly, the last relation implies

$$\frac{n^{n+1}}{n!}\int_b^{+\infty} (ve^{-v})^n dv \leq (n+1)(be^{-b})^n \frac{n^n}{n!}.$$

Consequently, from Eq. (3) and Eq. (4), its follows that

$$\frac{n^{n+1}}{n!}\int_b^{+\infty} (ve^{-v})^n dv \leq (n+1)(be^{-b})^n \frac{n^n}{n!} = 0,$$

which proves Eq. (2) in the assertion. Then, let $A \in \omega$-$ORCP_n$, where $(A, D(A))$ is the genertor of C_0-semigroup of contractions, let $\lambda > 0$ and

$$R(\lambda; A) = (\lambda I - A)^{-1}.$$

Then the mapping $\lambda \to R(\lambda; A)$ is of class C^∞ on $(0,+\infty)$ and for each $x \in X$, each $t > 0$ and $n \in \mathbb{N}^*$, we have

$$(I - \frac{t}{n}A)^{-n-1}x - T(t)x = \frac{n^{n+1}}{n!}\int_0^{+\infty} (ve^{-v})^n [T(tv)x - T(t)x]dv, \tag{6}$$

we need to show that Eq. (6) holds, by the relationship between a semigroup and its resolvent [see Rauf *et.al.*] [7]], we have

$$R(\lambda; A)x = \int_0^{+\infty} e^{-\lambda s}T(s)xds. \tag{7}$$

It follows that $\lambda \to R(\lambda;A)$ is analytic from $(0,+\infty)$ to $L(X)$, and differentiating n-times both sides in Eq. (6) with respect to λ and putting $s = vt$, we obtain

$$(R(\lambda;A))^{(n)}x = (-1)^n t^{n+1} \int_0^{+\infty} v^n e^{-\lambda tv} T(tv)x dv$$

for $\lambda > 0$, $x \in X$ and $A \in \omega\text{-}ORCP_n$. On the other hand,

$$(R(\lambda;A))^{(n)} = (-1)^n n! R(\lambda;A)^{n+1},$$

and so substituting $\lambda = \frac{n}{t}$ in the relations above, we have

$$[(\frac{n}{t}R(\frac{n}{t};A)]^{n+1}x = \frac{n^{n+1}}{n!}\int_0^{+\infty}(ve^{-v})^n T(tv)x dv.$$

Taking $b = 0$ in Eq. (5), we conclude that

$$\frac{n^{n+1}}{n!}\int_0^{+\infty}(ve^{-v})^n dv = 1$$

for each $n \in \mathbb{N}^*$. Therefore Eq. (6) holds. Next is to show that $\{T(t);t \geq 0\}$ is equicontinuous for each $\alpha \in (0,1)$. Let $\alpha \in (0,1)$ and fix $\beta \in (0,\alpha)$. Since $\{T(t);t \geq 0\}$ is equicontinuous, for each $\epsilon > 0$ there exists $\delta(\epsilon) > 0$ such that

$$\|T(t) - T(s)\|_{L(X)} \leq \epsilon$$

for each $t,s \in [\beta, 1/\beta]$ with $|t - s| \leq \delta(\epsilon)$. On the other hand, for the very same $\epsilon > 0$, there exists $a = a(\epsilon)$ and $b = b(\epsilon)$ with $0 < a < 1 < b < +\infty$ and such that, for each $t \in [\alpha, 1/\alpha]$ and $v \in [a,b]$, we have $tv \in [\beta, 1/\beta]$ and $|t - tv| \leq \delta(\epsilon)$.
So

$$\|T(tv) - T(t)\|_{L(X)} \leq \epsilon \quad \textbf{(8)}$$

for each $t \in [\alpha, 1/\alpha]$, and $v \in [a,b]$. From both Eq. (6) and Eq. (8), we deduce

$$\|(I - \frac{t}{n}A)^{-n-1} - T(t)\|_{L(X)} \leq 2\frac{n^{n+1}}{n!}\int_0^a (ve^{-v})^n dv$$
$$+ \epsilon\frac{n^{n+1}}{n!}\int_0^a (ve^{-v})^n dv + 2\frac{n^{n+1}}{n!}\int_b^{+\infty}(ve^{-v})^n dv$$

for each $n \in \mathbb{N}^*$. By the earlier assertions in the proof, it follows that

$$\lim_{n\to\infty} \sup \|(I - \frac{t}{n}A)^{-n-1} - T(t)\|_{L(X)} \leq \epsilon$$

for each $\epsilon > 0$. Consequently, for each $\alpha \in (0,1)$, we have

$$\lim_{n\to\infty}(I - \frac{t}{n}A)^{-n-1} = T(t) \quad \textbf{(9)}$$

in the norm $\|.\|_{L(X)}$, uniformly for $t \in [\alpha, 1/\alpha]$. In order to conclude the proof, we need to show that

$$\lim_{n\to\infty}(I - \frac{t}{n+1}A)^{-n-1} = T(t) \quad \textbf{(10)}$$

in the norm $\|.\|_{L(X)}$, uniformly for $t \in [\alpha, 1/\alpha]$. From Eq. (9) we deduce that, for each sequence $(a_n)_n \in \mathbb{N}$ of functions from $\mathbb{N}^* \cup \{0\} \to \mathbb{R}_+{}^*$ satisfying $\lim_{n\to\infty} a_n = t$ uniformly on each compact subset in $\mathbb{R}_+{}^*$, we have

$$\lim_{n\to\infty}(I - \frac{a_n(t)}{n}A)^{-n-1} = T(t)$$

in the norm $\|.\|_{L(X)}$, uniformly on each compact set in $\mathbb{R}_+{}^*$. This simply follows from the fact that, for each $\alpha \in (0,1)$, the set of functions $\{t \to (I - \frac{t}{n}A)^{-n-1}; n \in \mathbb{N}^*\}$ is equicontinuous on $[\alpha, 1/\alpha]$ because it is relatively compact in the space $C([\alpha, 1/\alpha]; L(X))$. Taking $a_n(t) = \frac{nt}{n+1}$, we obtain Eq. (10). Sufficiently by Eq. (6) its follows that $t \to (I - \frac{t}{n}A)^{-n}$ is continuous from $(0,+\infty)$ to $L(X)$ with respect to the norm $\|.\|_{L(x)}$. Since for each $\alpha \in (0,1)$, we have

$$\lim_{n\to\infty}(I - \frac{t}{n}A)^{-n} = T(t)$$

in the usual sup-norm topology of $C([\alpha, 1/\alpha]); L(X))$, its follows that the semigroup is continuous from $(0,+\infty)$ to $L(X)$, that is equicontinuos which complete the proof. ■

Theorem 3: Let $(A, D(A))$ be the generator of a C_0-semigroup of contraction $\{T(t);t \leq 0\}$, where $A \in \omega\text{-}ORCP_n$. Then $\{T(t);t \leq 0\}$ is compact if and only if

(i) $\{T(t);t \leq 0\}$ is equicontinuos, and

(ii) for each $\lambda > 0$, the operator $(\lambda I - A)^{-1}$ is compact.

Proof: Suppose $\{T(t);t \leq 0\}$ is a compact C_0-semigroup of contractions, let $\lambda > 0$, and $\lambda > 0$ be such that $t \to \lambda > 0$. Then, for each $\epsilon > 0$, there exists a finite family $\{x_1, x_2, \ldots, x_k(\epsilon)\}$ in $B(0,1)$ such that, for each $x \in B(0,1)$, there exists $i \in \{1,2,\ldots,k(\epsilon)\}$ with

$$\|T(t-\lambda)x - T(t-\lambda)xi\| \leq \epsilon.$$

Since the family $\{T(\cdot)xi; i = 1,2,\ldots,k(\epsilon)\}$ of continuous function from $[0,\infty)$ to X is finite, it is equicontinuous at t. Therefore, for every $\epsilon > 0$, there exists $\delta(\epsilon) \in (0,\lambda)$, such that

$$\|T(t+h)xi - T(t)xi\| \leq \epsilon$$

for each $i = 1,2,\ldots,k(\epsilon)$, and each $h \in \mathbb{R}$ with $|h| \leq \delta(\epsilon)$. We have

$$\|T(t+h)x - T(t)x\| \leq \|T(t+h)x - T(t+h)xi\|$$
$$+ \|T(t+h)xi - T(t)xi\| + \|T(t)xi - T(t)x\|$$
$$= \|T(\lambda+h)(T(t-\lambda)x - T(t-\lambda)xi)\|$$
$$+ \|T(t+h)xi - T(t)xi\| + \|T(\lambda)(T(t-\lambda)xi - T(t-\lambda)x)\|$$
$$\leq \|T(\lambda+h)\|_{L(X)}\|T(t-\lambda)x - T(t-\lambda)xi\|$$
$$+ \|T(t+h)xi - T(t)xi\| + \|T(\lambda)\|_{L(X)}$$
$$\|T(t-\lambda)xi - T(t-\lambda)x\| \leq 3\epsilon$$

for each $x \in B(0,1)$, and each $h \in \mathbb{R}$ with $|h| \leq \delta(\epsilon)$. Consequently

$$\|T(t+h) - T(t)\|_{L(X)} \leq 3(\epsilon)$$

for each $h \in \mathbb{R}$ with $|h| \leq \delta(\epsilon)$, which proves (i).

To prove (ii), let us recall that for each $\lambda > 0$, $x \in X$ and $A \in \omega\text{-}ORCP_n$,we have

$$(\lambda I - A)^{-1}x = R(\lambda;A)x = \int_0^\infty e^{-s\lambda}T(s)xds.$$

Let $\epsilon > 0$ and define $R_\epsilon(\lambda;A) : X \to X$ by $R_\epsilon(\lambda;A)x = \int_0^\infty e^{-s\lambda}T(s)xds$ for each $x \in X$ and $A \in \omega\text{-}ORCP_n$. We shall show that $R_\epsilon(\lambda;A)$ is a compact operator and also

$$\lim_{n\to\infty} \|R(\lambda;A) - R_\epsilon(\lambda;A)\|_{L(X)} = 0. \quad \textbf{(11)}$$

Let us observe that

$$\begin{aligned} R_\epsilon(\lambda;A)x &= T(\epsilon)\int_\epsilon^\infty e^{-s\lambda}T(s-\epsilon)xds \\ &= e^{-\lambda\epsilon}T(\epsilon)\int_0^\infty e^{-\tau\lambda}T(\tau)xd\tau \\ &= e^{-\lambda\epsilon}T(\epsilon)R(\lambda;A)x \end{aligned}$$

for each $x \in X$ and $A \in \omega\text{-}ORCP_n$. Since $R(\lambda;A)$ is linearly continuous and $e^{-\lambda\epsilon}T(\epsilon)$ is compact, it follows that $R_\epsilon(\lambda;A)$ is compact. On the other hand,

$$\|R(\lambda;A) - R_\epsilon(\lambda;A)\|_{L(X)} \le \int_0^\epsilon e^{\lambda t}\|T(t)\|_{L(X)}dt \le \frac{I-e}{-\lambda\epsilon}\lambda$$

for each $\epsilon > 0$ which proves (ii). It follows that $R(\lambda;A)$ is compact and sufficience, let $\{T(t);t \le 0\}$ be C_0-semigroup of contractions satisfying (i) and (ii). For $t > 0$ and $\lambda > 0$, we define $T_\lambda(t) : X \to X$ by

$$T_\lambda(t)x = \lambda R(\lambda;A)T(t)x$$

for each $x \in X$ and $A \in \omega\text{-}ORCP_n$. We want to prove that $T_\lambda(t)$, which obviously is compact, satisfies

$$\lim_{\lambda\to\infty} \|T_\lambda(t) - T(t)\|_{L(X)} = 0 \quad \textbf{(12)}$$

and

$$\begin{aligned} \|T_\lambda(t) - T(t)\|_{L(X)} &= \|\lambda\int_0^\infty e^{-\lambda\tau}(T(t+\tau) - T(t))d\tau\|_{L(X)} \\ &\le \lambda\int_0^\infty e^{-\lambda\tau}\|T(t+\tau) - T(t)d\tau\|_{L(X)}d\tau. \end{aligned} \quad \textbf{(13)}$$

Since the semigroup $\{T(t);t \ge 0\}$ is equicontinuous for each $\epsilon > 0$ there exists $\delta = \delta(\epsilon) > 0$ such that, for each $\tau \in (0,\delta)$, we have

$$\|T(t+\tau) - T(t)\|_{L(X)} \le \epsilon. \quad \textbf{(14)}$$

From Eq. (13) and Eq. (14), it follows that

$$\begin{aligned} &\|T_\lambda(t) - T(t)\|_{L(X)} \le \lambda\int_0^\delta e^{-\lambda\tau}\|T(t+\tau) - T(t)\|_{L(X)}d\tau \\ &+ \lambda\int_\delta^\infty e^{-\lambda\tau}(\|T(t+\tau)_{L(X)} + \|T(t)\|_{L(x)})d\tau \\ &\le (1-e^{-\lambda\delta})\epsilon + 2e^{-\lambda\delta}. \end{aligned} \quad \textbf{(15)}$$

Moving to the sup-limit for λ tending to $+\infty$ in Eq. (15), we obtain

$$\lim_{\lambda\to\infty} \sup\|T_\lambda(t) - T(t)\|_{L(X)} \le \epsilon$$

for each $\epsilon > 0$ and this relation implies Eq. (12) which ensures the compactness of the semigroup $\{T(t);t \ge 0\}$. Hence the proof is complete. ■

Theorem 4: Let $A : D(A) \subseteq X \to X$ be the infinitesimal generator of a C_0-semigroup of contractions $\{T(t);t \ge 0\}$ for which there exists $\lambda > 0$ such that $R(\lambda;A)$ is compact, then X is seperable. In particular, if the semigroup generated by A is compact where $A \in \omega\text{-}ORCP_n$, then X is separable.

Proof: Let us observe that, by virtue of the resolvent equation (see Rauf *et.al.*), we deduce that for each $\lambda > 0$, $R(\lambda;A)$ is compact. Let $(\lambda_n)_{n\in\mathbb{N}^*}$ be sequence of numbers, strictly decreasing to 0 and $n \in \mathbb{N}^*$ be arbitrary, provided $R(\lambda_n;A)$ is compact, then there exists a finite family D_n in $B(0,n)$ such that for each $x \in B(0,n)$, there exists $x_n \in D_n$ with

$$\|\lambda_n R(\lambda_n;A)x - \lambda_n R(\lambda_n;A)x_n\| \le \lambda_n. \quad \textbf{(16)}$$

Let $x \in X$, $A \in \omega\text{-}ORCP_n$ and $\epsilon > 0$, so that there exists $n \in \mathbb{N}^*$ such that

$$\begin{cases} \lambda_n \le \epsilon \\ \|x\| \le n \\ \|\lambda_n R(\lambda;A)x - x\| \le \epsilon. \end{cases}$$

Taking $x_n \in D_n$ satisfying Eq. (16), we deduce

$$\begin{aligned} \|x - \lambda_n R(\lambda_n;A)x_n\| \le &\|x - \lambda_n R(\lambda_n;A)x\| \\ &+ \|\lambda_n R(\lambda_n;A)x - \lambda_n R(\lambda_n;A)x_n\| \\ \le &2\epsilon. \end{aligned} \quad \textbf{(17)}$$

Inequality Eq. (17) shows that the set $D = \cup_n \lambda_n R(\lambda_n;A)D_n$ is dense in X. Since this set is a countable union of finite sets, then it is countable too, and therefore X is separable. Finally, if the semigroup generated by A is compact, there exists $\lambda > 0$ such that $R(\lambda;A)$ is compact, which complete the proof. ■

4. Conclusions

In this paper, we have been able to established that $\omega-$Order reversing partial contraction mapping possesses some characteristics of compact semigroup of linear operator by considering its closeness, linear and equiontinuous nature.

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