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Individual Behavior on Taxation: An Optimization Model

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Abstract**:** This paper aims to propose a basic optimization model in making decision on consumption utility that is used by economists to explain individuals' behavior on taxation. This model assumes that individuals who are constrained by limited incomes will behave as if they are using their purchasing power to achieve the most noteworthy utility conceivable. In addition to this, the purchasing power is dependent on the individuals' limited income. Upon the consumption utility, the optimization model is formulated and the decision variables are identified. During the calculation procedure, the Lagrange multiplier method is applied to solve the optimization model, where the Excel solver is employed as a computational tool. Technically, the results show that individuals are able to accept the decision outcome as their utility is maximized over the budget constraint. As a result, to maximize utility, individuals will choose a bundle of commodities for which the rate of trade-off between any two goods is equivalent to the proportion of the good's business sector costs. On the other hand, market costs pass on information about circumstance costs to individuals, and this information is an essential part in influencing the decisions that are made. In conclusion, an alternative decision making by the individuals to make the right decision on consumption utility is provided.

Keywords**:** budget constraint, consumption utility, Lagrange multiplier method, optimization, taxation.

1. Introduction

Individual consumption is restricted by the purchasing power which is depending on the individual income. In fact, an individual with a high income has a best purchasing power, while another individual with a low income has a limited purchasing power. However, the taxation is taken on each purchase, where any individual must pay the consumption tax for the purchase made. Due to the taxation on the purchase and the limited income, individuals would think of maximizing the purchase choice in making purchase decision. In economics, the maximization of the purchase choice is referred to as the maximization of the utility [1]. Here, the utility is a measure of the individuals' preferences over a set of goods and services that are consumed in a certain period of time [2]. This measure of the preference is declared as unique only up to an order-preserving transformation. Simply, this measure of preference is a basic model to explain the individual behavior, where the utility is a maximized subject to a budget constraint. Therefore, the individual behavior on the utility for the consumption on goods and services could be examined.

Actually, a taxpayer utility function has been examined by a set of variables which are consumption, labor and tax-evasion propensity [3]. Accordingly, a main framework is presented to analyze the taxpayer's decision making process. In addition to this, the framework is working under an assumption that there are two reduction methods in the economy. The first method is using the tax optimization techniques, such that the tax burden may decrease, but still fully comply with binding laws. However, it may generate some transactional costs. The second method is related to fiscal fraudulent which is an alternative method in reducing the tax due with no direct transaction costs and involving tax litigation risk. By virtue of this, an optimization problem is presented to handle the individual taxpayer who is trying to reduce tax rate on his/her income [3].

A model of taxpayer's decision on evade from the tax is developed [4]. The problem of the model developed is that utility maximization is restricted under the condition of uncertainty. The taxpayer's decision is considered as a portfolio allocation problem. They must decide on how much of their income to be taxed. To avoid any risk, they need to report their full income. However, some of the taxpayers only report a surface of their income. This is categorized as a tax practitioner's ethical behaviour.

The motivation of this paper is from Lin [5]. His paper has analysed the effects of China's upcoming value-added tax (VAT) reform of removing investment from the tax base on capital accumulation and the welfare of the rich and the poor [5]. However, some modification has been made such as method and the values of rates.

This paper is organized as follows. Section 2 discusses on the problem statement. Section 3 gives a detailed explanation about the method. Section 4 shows analytical and simulation results. Section 5 makes a conclusion of the paper.

2. Problem Statement

Suppose that in period t, the output per person Y_t is defined by $Y_t = F\left(K_t, L_t\right)$, where K_t is the average physical capital owned by each person in period t and L_t is the amount of labor supplied per young person in period t. Let $y_t = Y_t / L_t$ be the output-labour ratio, we have

$$y_t = f(k_t),\ f'(k_t) > 0,\ f''(k_t) < 0 \tag{1}$$

where $k_t = K_t / L_t$ is the ratio of capital to the effective labour. After one period of production, it is assumed that the capital is fully depreciated. The marginal product is the effect of the rate of return to the market factor, which is absolutely competitive given by

$$1 + r_t = f'(k_t) \tag{2}$$

$$w_t = f(k_t) - k_t f'(k_t) \tag{3}$$

where $1 + r_t$ is the rate of return on capital or the interest rate in period t, and w_t is the rate of return to labour, which is the wage rate.

Now, consider the optimization model for individual behavior on taxation, where rich and poor individuals are taken into account. The objective function of the model is to maximize the utility for rich and poor individuals subject to their respective budget constraints. This model is also assumed that the individuals receive wage income in the first period of life, whereas capital income is received in the second period. With the assumption that the utility function is the constant elasticity of substitution (CES) type, this optimization model is given by

$$\text{Maximize } u(c_{i,t}, c_{i,t+1}) = \frac{(c_{i,t})^\delta + \rho_i (c_{i,t+1})^\delta}{\delta}$$

subject to (4)

$$c_{i,t}(1+\phi) \le w_t e_i (1-\pi_t) - (1+\chi) S_{i,t} + \tau_{i,t} c_{i,t+1}(1+\phi) \le (1+r_{t+1}) S_{i,t}$$

For convenience, the budget constraint is written by

$$c_{i,t}(1+\phi) + \frac{c_{i,t+1}(1+\phi)}{1+r_{t+1}} = w_t e_i (1-\pi_t) - \chi S_{i,t} + \tau_{i,t} \tag{5}$$

where $c_{i,t+j}$ is consumption in period $t+j$ of an individual of type i, for $i = 1, 2$, born in period t, which is called generation t, $j = 0, 1$; $0 < \delta \le 1$; $\rho_i < 1$ is the constant pure rate of time preference of type i individual; ϕ is the tax rate on consumption; χ is the average tax rate on investment, which is equal to savings in equilibrium; π_t is the average tax rate on labour in period t; $\tau_{i,t}$ are transfers, which are taxes if negative, in the first period of life of an individual born in period t. Total amount of value-added tax paid is consumption tax plus investment tax, which is a savings tax $S_{i,t}$ in period t for $i = 1, 2$.

It is highlighted that the changes of the model (4) are investment being removed from tax base and the tax rate on consumption being endogenous. Essentially, there are the firm, the consumer, and the government sectors in the economy. This model only focuses on the rich (type 1 agent) and the poor (type 2 agent). Both rich and poor agents actively invest with e_i units of effective labour in the first time frame. It is assumed that the rich agent has more endowment than the poor agent when $e_1 > e_2$. Notice that $\alpha_1 + \alpha_2 = 1$, where α_i stands for the proportion of type i individual in the total population. Therefore, the quantity of rich agents is $\alpha_1 N$ and the quantity of poor agents is $\alpha_2 N$ for the N individuals in the population that does not grow in each period.

3. Lagrange-Multiplier Method

Lagrange multipliers method is important in economics. It is a marginal valuation or shadow price of the scarce resources [6]. Thus, the utility maximization problem given in (4) is solved by the Lagrange multiplier method. From (4), the Lagrangian function is defined by

$$\Pi = \frac{(c_{i,t})^\delta + \rho_i (c_{i,t+1})^\delta}{\delta} + \mu_t \left[E_{i,t} - c_{i,t}(1+\phi) - \frac{c_{i,t+1}(1+\phi)}{1+r_{t+1}} \right] \tag{6}$$

where μ_t is the Lagrange multiplier, and

$$E_{i,t} = w_t e_i (1-\pi) - \chi S_{i,t} + \tau_{i,t} \tag{7}$$

is the first-period disposable income. The first order conditions are derived as follow:

$$\frac{\partial \Pi}{\partial c_{i,t}} = (c_{i,t})^{\delta-1} - \mu_t (1+\phi) = 0 \tag{8}$$

$$\frac{\partial \Pi}{\partial c_{i,t+1}} = \rho_i (c_{i,t+1})^{\delta-1} - \frac{\mu_t (1+\phi)}{1+r_{t+1}} = 0 \tag{9}$$

$$\frac{\partial \Pi}{\partial \mu_t} = E_{i,t} - c_{i,t}(1+\phi) - \frac{c_{i,t+1}(1+\phi)}{1+r_{t+1}} = 0 \tag{10}$$

From (8),

$$(c_{i,t})^{\delta-1} = \mu_t (1+\phi) \tag{11}$$

From (9),

$$\rho_i (c_{i,t+1})^{\delta-1} = \frac{\mu_t (1+\phi)}{1+r_{t+1}} \tag{12}$$

Substitute (11) into (12) to yield

$$(c_{i,t}) = \left[(1+r_{t+1}) \rho_i \right]^{\frac{1}{\delta-1}} c_{i,t+1} \tag{13}$$

From (10) and (13), the first-period disposable income becomes

$$E_{i,t} = c_{i,t}(1+\phi) + \frac{c_{i,t+1}(1+\phi)}{1+r_{t+1}}$$

$$= \left[(1+r_{t+1}) \rho_i \right]^{\frac{1}{\delta-1}} c_{i,t+1}(1+\phi) + \frac{c_{i,t+1}(1+\phi)}{1+r_{t+1}} \tag{14}$$

After some algebraic manipulations, we obtain

$$c_{i,t+1}(1+\phi) = \frac{E_{i,t}}{\left[(1+r_{t+1}) \rho_i \right]^{\frac{1}{\delta-1}} + (1+r_{t+1})^{-1}} \tag{15}$$

Consider (7) in (15), gives

$$c_{i,t+1} = \frac{w_t e_i\left(1-\pi_t\right) - \chi S_{i,t} + \tau_{i,t}}{\left\{\left[\left(1+r_{t+1}\right)\rho_i\right]^{\frac{1}{\delta-1}} + \left(1+r_{t+1}\right)^{-1}\right\}\left(1+\phi\right)} \tag{16}$$

Then, (13) can be written by

$$c_{i,t} = \frac{w_t e_i\left(1-\pi_t\right) - \chi S_{i,t} + \tau_{i,t}}{\left[1+\left(1+r_{t+1}\right)^{\frac{\delta}{1-\delta}}\left(\rho_i\right)^{\frac{1}{1-\delta}}\right]\left(1+\phi\right)} \tag{17}$$

Consider the second period budget constraint and (16) in the following equation

$$S_{i,t} = c_{i,t+1}\frac{1+\phi}{1+r_{t+1}} \tag{18}$$

By merging (16) into (18) and doing some algebraic manipulations we obtain

$$S_{i,t} = \frac{e_i\left(1-\pi_t\right)w_t + \tau_{i,t}}{1+\chi+\left(1+r_{t+1}\right)^{\frac{\delta}{\delta-1}}\left(\rho_i\right)^{\frac{1}{\delta-1}}} \tag{19}$$

where $S_{i,t}$ represents savings in period t of a type i agent born in period t, which is the difference between the first period income and the first-period consumption. Here, equation (19) indicates savings $S_{i,t}$ depending on the interest rate, the wage rate, the tax rates on wage income and investment, transfers, the rate of time preference and other parameters.

4. Results and Discussion

The optimization model given in (4) and (5) is solved numerically by taking the following values of parameters: $\beta = 0.7; A = 5; \delta = 0.5; e_1 = 8; e_2 = 4; \alpha_1 = \alpha_2 = 0.5; g = 5$

In addition, Table 1 shows some additional information on rates in order to solve the model. The information includes; tax rate on consumption ϕ, 0.01; reducing the transfers $\tau_{i,t}$ from 3.00 to 2.07; reducing the investment tax rate χ, from 0.15 to 0.10; reducing the interest rate r from 0.0997 to 0.0993; increasing the wage rate w, from 3.9980 to 3.9987; constant rate of the time preference of the agents ρ 0.8; and increasing the labour income tax rate π, from 0.04 to 0.06.

From Table 2, it is shown that the savings of the rich S_1, is decreasing from 13.11 to 12.75. The same goes to the savings for the poor S_2 which is also decreasing from 6.77 to 6.42. However, with the decreasing rate of investment, the utility for the poor u_2 is increasing from 8.75 to 9.12, while the utility for the rich u_1 is decreased. Besides, the consumption of the rich agent in period 1, $c_{1,1}$ and in period 2, $c_{1,2}$ are both decreasing. Consumption of the poor agent in period 1, $c_{2,1}$ is increased from 6.59 to 7.16, while in consumption period 2, $c_{2,2}$ is also increased from 5.099 to 5.54.

Table 1. Additional information on rates

Item	Value
Tax rate on consumption, ϕ	0.01
Transfers, $\tau_{i,t}$	3.00 2.07
The average tax rate on investment, χ	0.15 0.10
The interest rate, r_t	0.0997 0.0993
The wage rate, w_t	3.9980 3.9987
The constant pure rate of time preferences, $\rho_i < 1$	0.8 0.8
The average tax rate on labour in period, π	0.04 0.06

Even though the rate of investment and interest rate are decreased, the labor income tax rate and the wage rate are still increased. That is, the reason why the utility for the rich u_1 is decreasing from 14.63 to 14.43. The impact of the increasing in labor income tax rate and wage rate is higher to the rich agent than the poor agent. It is because majority of the rich agents receive higher salary.

Table 2. The simulation result of the tax rate reform on individual utility

S_1	S_2	$c_{1,1}$	$c_{1,2}$	$c_{2,1}$	$c_{2,2}$	u_1	u_2
13.11	6.77	18.44	14.27	6.59	5.099	14.63	8.75
12.75	6.42	17.94	13.87	7.16	5.54	14.43	9.12

From the information, it might be helpful for the rich and poor agents in making decision on their consumption of utility. The both agents can behave with their limitation in income to make a choice in their life. In addition, there are also other economic behavioral factors that influence the agents in making decision to maximize their utility in taxation. There are other economic behavioral factors such as tax evasion and tax compliance. To face the factors, the individuals should seriously look through the Chinese ethical behavior in implementing the individual tax compliance [7].

5. Conclusion

In this paper, the basic optimization model was used to examine the individuals' behavior on taxation. The economists used to optimize the individuals' consumption utility in order to make a decision. This paper has defined the rich and poor agent. It is believed that the individuals' purchasing power is constrained by their limited income. The optimal results of their utility is over the budget constraint and they are able to accept the decision outcome. Market information in this paper such as the tax rate on investment, the interest rate, the wage rate, the capital–labour ratio, the output–labour ratio, present value of taxes paid by the rich and the poor, and the labour income tax rate are assumed as essential parts in influencing the decisions is really made. Some other factors that influence the decision like tax evasion and tax compliance also have been discussed in this paper. It is concluded that this paper contributes an alternative decision making to the individuals to make the right decision on consumption utility. This extension on taxation optimization is left for further research.

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