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Fuzzy Analytic Hierarchy Process (FAHP) with Cosine Consistency Index for Coastal Erosion Problem: A Case Study of Setiu Wetlands

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Abstract: Decision making is the process of selecting the best choice out of multiple alternatives and multiple factors to be considered. Decision making which involves human judgments is always imprecise and subjective. Fuzzy AHP has been introduced to cater those deficiencies in traditional AHP. One of the vital issues in Fuzzy AHP is the consistency measure. The studies in that problem has been explored by many scholars, however, many applications still ignore the consistency measure in their works. Therefore, this paper aims to introduce the Cosine Consistency Index (CCI) proposed by Kou and Lin into Fuzzy AHP. The proposed methodology is applied to the case study of Setiu Wetlands coastal areas in Setiu, Terengganu to weight the factors of coastal erosion. The obtained results show that land use/cover change (17.4%) is the most contributing factor towards the coastal erosion. It is followed by sea area class (14%), erodibility (11%), population density (10.1%), shoreline evolution (9.9%), sedimentary discharge (8.7%), tidal range (7.2%), storm surge (7.1%), wave height (4.9%), budgetary revenue of local government (3.6%), relative sea level (3.3%) and coastal protection (2.8%). The proposed methodology is applicable to many other decision making problems. The applied cosine consistency index into the Fuzzy AHP evaluation process makes the obtained results more plausible and convincing.

Keywords: Fuzzy-AHP, cosine consistency index (CCI), coastal erosion.

1. Introduction

Fuzzy AHP is one of the popular methodologies in Multi Criteria Decision Making (MCDM) where the fuzzy set theory and the AHP method are combined. Fuzzy AHP has been established for solving real world problems especially when dealing with fuzziness and uncertainty. The applications of Fuzzy AHP have been widely seen in various fields like teaching performance evaluation [1], construction project [2], heart failure risk prediction [3], selection problems [4], water loss management [5], mineral prospectivity mapping [6], design concept evaluation [7], marine management and assessment [8-11], assessment of energy policies and technologies [12] and coastal erosion [13], just to cite few. According to a recent survey on fuzzy MCDM techniques [14], Fuzzy AHP is the second most widely used technique in a stand-alone model just after AHP. However, the final decision of any AHP related method is only plausible if the pairwise comparison matrices passed the consistency tests.

In the real world decision problems, the pairwise comparison matrices are seldom able to achieve a perfect consistency [15]. Consistency is closely related with the decision makers' rationality in providing preferences. If the number of element to be compared is large, the decision makers tend to confuse during the judgment elicitation and this can lead to an inconsistency of preferences. Besides that, the nature complexity of decision problems, limitations of scale used and inaccuracies due to lack of concentration during judgmental processes can also leads to an inconsistency. Inconsistency is an inevitable event, however it can be minimize in various ways. Inconsistency in judgement may contribute to unreliable weights and ranking orders for alternatives. Therefore, checking satisfactory consistency is necessary to ensure the rationality of the decisions.

Consistency during judgement process is one of the important issues in AHP approach. Many scholars have studied this issue and introduced several consistency indices to carry out this task; for instance the Consistency Index [16], the Harmonic Consistency Index [17], the Geometric Consistency Index [18], the statistical index by Lin *et al.* [19], and the induced matrix method [20] and many more. Choosing a right consistency index is very important in decision making processes as it can affect the decision outcome in real applications. The consistency check must pass a certain threshold assigned for each particular consistency index, or else the pairwise comparison matrix needs to be reviewed until the satisfactory consistency is achieved. On the other hand, the revising process is a tedious and time consuming process. Many studies have been carried out to improve the consistency level of pairwise comparison without having to return it back to decision makers [21-25]. These pass research shows the importance of consistency in AHP method. However, so far, little work has been found in literature on the consistency measure involving fuzzy numbers. Many applications of Fuzzy AHP have ignored the consistency measure step and proceed with priority weights finding [11, 26-29].

The issue of consistency in AHP using fuzzy numbers was first undertaken by Salo [30]. Instead of the fuzzy arithmetic approach, fuzzy weights using an auxiliary mathematical programming formulation describing relative fuzzy ratios were derived. Later on Leung and Cao [31] proposed a notion of tolerance deviation of fuzzy relative importance that is strongly related to Saaty's consistency ratio. From the literature review on consistency measure involving Fuzzy

AHP, it is found that most of the literature implemented the consistency index was proposed by Saaty [16]. The fuzzy numbers are defuzzified before implementing the consistency index. Burut *et al*. [32] proposed the centric consistency index which is extended from the geometric consistency index proposed by Crawfold [18]. Ramik and Korviny [33] proposed a new inconsistency index based on the distance to the specific consistent matrix corresponding to possibility theory. Mikhailov [34] proposed a consistency index for fuzzy preference programming prioritization method.

Among many existing consistency indices, the one that proposed by Kou and Lin [35] called cosine consistency index (CCI) is a good option to check the consistency of pairwise comparison matrices. This consistency index is based on cosine maximization method. CCI value can be obtained by calculating average similarity between priority vector and each column of pairwise comparison matrix with an objective of maximizing the CCI value. Kou and Lin [35] proposed this index to check the consistency level of crisp pairwise comparison matrix in AHP method. Hence, this paper aims to introduce CCI into Fuzzy AHP and applied the proposed methodology in coastal erosion risk assessment. The arrangement of this paper is as follows: section 2 discusses on a few preliminary definitions used in this study, section 3 presents the proposed methodology, section 4 discusses the implementation of the proposed methodology to the case study of coastal erosion and finally concluding remarks is in section 5.

2. Preliminaries

The fundamental theories used in this study are presented in this section.

2.1 Fuzzy Sets and its arithmetic operations

Fuzzy set theory was introduced by Zadeh [36] to deal with uncertainty and fuzziness information. The application of fuzzy set theory has been established to solve many real world problems. The definition of fuzzy set theory is shown in the Definition 1.

Definition 1. Let X be universe of discourse, $\tilde{A}$ is a fuzzy subset of X such that for all $x \in X$. $\mu_{\tilde{A}}(x) \in [0,1]$ which is assigned to stand for the membership of x to $\tilde{A}$, and $\mu_{\tilde{A}}(x)$ is called the membership function of set $\tilde{A}$.

Informally, fuzzy sets are the concept of a continuum of grades of membership ranging between zero and one. If the assigned value is zero, the element does not belong to the set and if the value assigned is one, then the element belongs completely to the set [37]. Lastly, the value which lies between 0 and 1 belongs to the fuzzy set only partially.

The commonly used fuzzy numbers are triangular fuzzy numbers and trapezoidal fuzzy numbers. The triangular fuzzy numbers is the generalized form of trapezoidal fuzzy numbers if the two most promising values of the trapezoidal fuzzy number are same. In addition, triangular fuzzy numbers have been used in many applications as its intuitive appeal and computational efficiency. Triangular fuzzy numbers are applied to deal with the fuzziness and vagueness that exist in the decision problem. The definition and its arithmetic operations are shown in definition 2.

Definition 2. A triangular fuzzy number of $\tilde{A}$ is represented as $\tilde{M} = (l, m, u)$ and its membership function is described as in (1).

$$\mu_{\tilde{M}}(x) = \begin{cases} 0, & x < l \\ \dfrac{x-l}{m-l}, & l \le x \le m \\ \dfrac{u-x}{u-m} & m \le x \le u \\ 1 & x > u \end{cases} \qquad (1)$$

The parameters *l,m,u*, indicate the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy amount respectively [37].

The following are the arithmetic operations of triangular fuzzy numbers [11]. Let $\tilde{M}_1 = (l_1, m_1, u_1)$ and $\tilde{M}_2 = (l_2, m_2, u_2)$

i. Addition: $(l_1 + l_2, m_1 + m_2, u_1 + u_2)$

ii. Subtraction: $(l_1 - l_2, m_1 - m_2, u_1 - u_2)$

iii. Multiplication: $(l_1 \times l_2, m_1 \times m_2, u_1 \times u_2)$

iv. Division: $\tilde{M}_1^{-1} = \left(\dfrac{1}{u_1}, \dfrac{1}{m_1}, \dfrac{1}{l_1}\right)$ (2)

2.2 Assessment Model

In the following, the outlines of the Chang's extent analysis method [4] on Fuzzy AHP are given:

Let, $U = \{u_1, u_2, \ldots, u_m\}$ be a goal set and $X = \{x_1, x_2, \ldots, x_n\}$ be the object set. Each object is taken and extent analysis for every goal is performed, respectively. Therefore, m extent analysis values for each goal can be obtained, with the following signs:

$$\tilde{M}_{g_i}^1, \tilde{M}_{g_i}^2, \ldots \tilde{M}_{g_i}^m, \qquad i = 1, 2, \ldots, n \qquad (3)$$

where all the $\tilde{M}_{g_i}^j$ (j=1,2,…,m) are triangular fuzzy numbers (TFNs) and g_i is the corresponding goal. The value of fuzzy synthetic extent with respect to the i^{th} object is defined as

$$S_i = \sum_{j=1}^{m} \tilde{M}_{g_i}^j \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} \tilde{M}_{g_i}^j\right]^{-1} \qquad (4)$$

To obtain $\sum_{j=1}^{m} \tilde{M}_{g_i}^j$, the fuzzy addition operation of m extent analysis values is performed such as

$$\sum_{j=1}^{m} \tilde{M}_{g_i}^j = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right) \qquad (5)$$

In order to obtain $\left[\sum_{i=1}^{n} \sum_{j=1}^{m} \tilde{M}_{g_i}^j\right]^{-1}$, the fuzzy addition operation of $\tilde{M}_{g_i}^j$ (j = 1, 2, . . . ,m) values is carried out as below.

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\tilde{M}_{g_i}^{j}=\left(\sum_{i=1}^{n}l_i,\sum_{i=1}^{n}m_i,\sum_{i=1}^{n}u_i\right) \quad (6)$$

and then the inverse of the vector in (7) is computed such that

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\tilde{M}_{g_i}^{j}\right]^{-1}=\left(\frac{1}{\sum_{i=1}^{n}u_i},\frac{1}{\sum_{i=1}^{n}m_i},\frac{1}{\sum_{i=1}^{n}l_i}\right) \quad (7)$$

The degree of possibility of $\tilde{M}_2=(l_2,m_2,u_2)\geq\tilde{M}_1=(l_1,m_1,u_1)$ is defined as

$$V(\tilde{M}_2\geq\tilde{M}_1)=\sup_{y\geq x}\left[\min\left(\mu_{\tilde{M}_1}(x),\mu_{\tilde{M}_2}(y)\right)\right] \quad (8)$$

and it can be expressed as follows.

$$V(\tilde{M}_2\geq\tilde{M}_1)=\begin{cases}1 & if\ m_2\geq m_1\\ 0 & if\ \ l_1\geq u_2\\ \dfrac{l_1-u_2}{(m_2-u_2)-(m_1-l_1)} & otherwise\end{cases} \quad (9)$$

The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers $\tilde{M}_i(i=1,2,\ldots,k)$ can be defined by

$$V\left(\tilde{M}\geq\tilde{M}_1,\tilde{M}_2,\ldots,\tilde{M}_k\right)=\min V\left(\tilde{M}\geq\tilde{M}_i\right) \quad (10)$$

Assume that

$$d'(A_i)=\min V(S_i\geq S_k) \quad (11)$$

for $k=1,2,\ldots,n \,\&\, k\neq i$, then the weight vector is given by

$$W'(A_i)=\left(d'(A_1),d'(A_2),\ldots,d'(A_n)\right)^T \quad (12)$$

where $A_i(i=1,2,\ldots,n)$ are n elements.

Via normalization, the normalized weight vectors are

$$W=\left(d(A_1),d(A_2),\ldots,d(A_n)\right)^T \quad (13)$$

where W is a non-fuzzy number.

2.3 Cosine Consistency Index

The following theorems are implemented to check for consistency of all pairwise comparison matrices in this study.

Theorem 1. [35] Let C^* be the optimal objective function value of an optimization model. Then

$$C^*=\sqrt{\sum_{i=1}^{n}\left(\sum_{j=1}^{n}b_{ij}\right)^2} \quad (14)$$

where $b_{ij}=a_{ij}\Big/\sqrt{\sum_{k=1}^{n}a_{kj}^2}>0$ (15)

3. The Proposed Fuzzy AHP with Cosine Consistency Index (CCI)

The proposed Fuzzy AHP model to assess the coastal erosion risk factors is composed in a systematic flow of framework shown in Fig. 1.

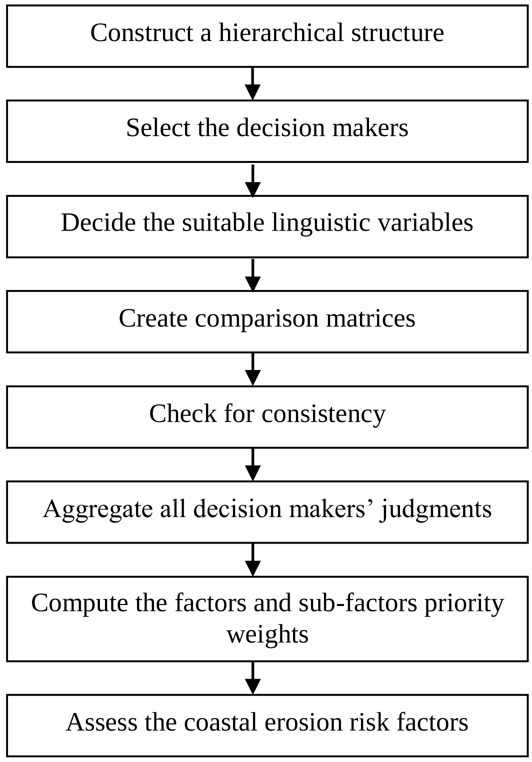

Figure 1. The methodology framework

Step 1. Construct a hierarchical structure of the problem

Developing a hierarchical structure is the most crucial step in AHP approach in which can provide a clear representation of the whole problem. The hierarchical structure consists of main goal, factors, sub-factors and alternatives.

Step 2. Select the decision makers

A group of decision makers is formed which consist of the experts from the related field. In order to obtain a reliable result, it is important to consider the decision makers' background. The decision makers must be someone who has experience with the research topic as each of them needs to give judgment in the evaluation process. The relative weights of factors and sub-factors are then obtained from the decision makers' judgments.

Step 3. Decide the suitable linguistic variables

Linguistic variables are applied to describe the relative importance of factors and sub-factors. In addition, they are able to express words or sentences of human language. The evaluation process is done through questionnaires which are in the form of linguistic variables. In order to proceed with mathematical operations, linguistic variables should be converted into fuzzy scales. In AHP approach, the nine-point ratio scale is used to perform pair-wise comparison while in this study, triangular fuzzy numbers proposed by Kahraman *et al.* [38] are used to represent fuzzy pair-wise comparisons. The triangular fuzzy numbers scale is demonstrated in Table 1.

Table 1. Triangular Fuzzy Numbers

Fuzzy number	Linguistic scale	Membership function	Inverse
1	Equally important	$(1,1,1)$	$(1,1,1)$
3	Moderately important	$(1,3/2,2)$	$(1/2,2/3,1)$
5	More important	$(3/2,2,5/2)$	$(2/5,1/2,2/3)$
7	Strongly important	$(2,5/2,3)$	$(1/3,2/5,1/2)$
9	Extremely important	$(5/2,3,7/2)$	$(2/7,1/3,2/5)$

Step 4. Create comparison matrices

Pair-wise comparison matrices are constructed to transform the linguistics variables into triangular fuzzy numbers. A pair-wise comparison matrix can be expressed as:

$$\tilde{A} = \left(\tilde{a}_{ij}\right)_{n\times n} = \begin{bmatrix} 1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & 1 \end{bmatrix} \quad (16)$$

where $\tilde{a}_{ij}$ is a triangular fuzzy number after comparing factors *i* to factor *j* while $\left(\tilde{a}_{ij}\right)^{-1}$ is a triangular fuzzy number comparing factor *j* to factor *i*. $\tilde{a}_{ij}$ and $\left(\tilde{a}_{ij}\right)^{-1}$ can be denoted as below.

$$\tilde{a}_{ij} = \left(l_{ij}, m_{ij}, u_{ij}\right)$$
$$\left(\tilde{a}_{ji}\right) = \left(\tilde{a}_{ij}\right)^{-1} = \left(u_{ij}^{-1}, m_{ij}^{-1}, l_{ij}^{-1}\right)$$

Step 5. Check for consistency

Consistency needs to be measured to assure that the decision makers' judgments are reliable and also to avoid any misleading solutions. The fuzzy matrices are converted into crisp matrices by defuzzification methods [39]. There are a few defuzzification methods [37, 40, 41] that may be used and for this study, Chang *et al.*'s method [42] is used. The Chang *et al.*'s method can clearly display the fuzziness of the problem as the preference (α) and risk tolerance (λ) of decision makers are considered. The defuzzification of a triangular fuzzy number denoted as $\tilde{a}_{ij} = \left(l_{ij}, m_{ij}, u_{ij}\right)$ is as follows.

$$\left(a_{ij}^{\alpha}\right)^{\lambda} = \left[\lambda \cdot l_{ij}^{\alpha} + (1-\lambda)u_{ij}^{\alpha}\right] \quad 0 \le \lambda \le 1,\ 0 \le \alpha \le 1 \quad (17)$$

where $l_{ij}^{\alpha} = \left(m_{ij} - l_{ij}\right)\times\alpha + l_{ij}$, is the left-end value of α-cut for a_{ij}, $u_{ij}^{\alpha} = u_{ij} - \left(u_{ij} - m_{ij}\right)\times\alpha$, is the right-end value of α-cut for a_{ij}. The higher the α, the more stable the decision making environment and α of zero indicates the highest degree of uncertainty. In addition, λ shows the degree of a decision-maker's optimism. The decision-maker is highly optimistic when λ is 0 and vice versa.

The crisp matrices can be expressed as follows:

$$\left[\left(\tilde{A}^{\alpha}\right)^{\lambda}\right] = \left(\tilde{a}_{ij}{}^{\alpha}\right)^{\lambda} = \begin{bmatrix} 1 & \left(\tilde{a}_{12}^{\alpha}\right)^{\lambda} & \cdots & \left(\tilde{a}_{1n}^{\alpha}\right)^{\lambda} \\ \left(\tilde{a}_{21}^{\alpha}\right)^{\lambda} & 1 & \cdots & \left(\tilde{a}_{2n}^{\alpha}\right)^{\lambda} \\ \vdots & \vdots & \ddots & \vdots \\ \left(\tilde{a}_{n1}^{\alpha}\right)^{\lambda} & \left(\tilde{a}_{n2}^{\alpha}\right)^{\lambda} & \cdots & 1 \end{bmatrix} (18)$$

The cosine consistency index for the pairwise comparison matrices can be computed using CCI equation in Theorem 1.

According to Kou and Lin [36] the pairwise comparison matrix is perfectly consistent if $C^* = n$. On the contrary, if $0 < C^* < n$, the pairwise comparison matrix is inconsistent. In order to eliminate the influence of the size of a pairwise comparison matrix, the objective function value C^* is divided by *n*, where C^*/n is called cosine consistency index (CCI) of the pairwise comparison matrix and takes on values in the interval (0, 1]. CCI of 1 indicates a perfect consistency or otherwise, if CCI value less than zero, it is considered as inconsistent. Kou and Lin [36] gives a general rule to accept a pairwise comparison matrix which is the CCI must be at least 90% or else the decision makers need to review their judgments.

Step 6. Aggregation of decision makers' judgments

Aggregation of individual decision maker's judgments is necessary to achieve a consensus group decision. There are two commonly used aggregation operators for AHP method which are aggregation of individual judgments (AIJ) and aggregation of individual priorities (AIP) [40]. These approaches can also be employed in the Fuzzy AHP. In the AIJ approach, the group comparison matrix is obtained from the individual comparison matrices. The priorities are calculated from the new constructed group judgment matrices. However, in the AIP approach, the individual priorities are obtained beforehand, and then the group priorities are calculated. AIJ is calculated using geometric mean operations whereas AIP is calculated using arithmetic mean operations. This paper utilizes the AIJ approach to do the aggregation process. Let *R* is the number of decision-makers and *n* is number of elements. As a result of the pairwise comparisons, we get a set of *R* matrices, $\tilde{A}_r = \left\{\tilde{a}_{ijr}\right\}$, where $\tilde{a}_{ijr} = \left(l_{ijr}, m_{ijr}, u_{ijr}\right)$ represents a relative importance of element *i* to *j*, as assessed by the expert *r*. The triangular fuzzy numbers in the group judgment matrix can be obtained by using the following equation:

$$l_{ij} = \min(l_{ijr}) \qquad r = 1,2,\ldots,R$$
$$m_{ij} = \sqrt[R]{\prod_{r=1}^{R} m_{ijr}}$$
$$u_{ij} = \max(u_{ijr}) \qquad r = 1,2,\ldots,R \qquad (19)$$

Step 7. Compute the factors and sub-factors priority weights

The weights of factors and sub-factors can be obtained by performing the extend analysis Fuzzy AHP method from the group fuzzy pairwise comparison matrices.

4. Case Study

The proposed methodology is applied to find relative weights of coastal erosion risks factors. First, the factors and sub-factors of coastal erosion were identified which was adapted from Luo *et al.* [13]. Since Luo *et al.* [13] listed the criteria based on the case study in China, the factors and sub-factors for this study is finalize after some discussions with the experts. Table 2 shows the chosen factors and sub-factors for this study.

Table 2. List of criteria and sub-criteria of coastal erosion risk factors

Factor	Code	Sub-factor	Code
Geo-environmental Hazard	GEH	Shoreline evolution	SE
		Erodibility	E
		Storm surge	SS
		Tidal range	TR
		Relative sea-level	RSL
		Wave height	WH
		Sediment discharge	SD
Socio-economic Vulnerability	SEV	Population density	PD
		Coastal protection	CP
		Budgetary Revenue of Local Government	BRLG
		Sea area class	SAC
		Land use/cover change	LUCC

The hierarchical structure of coastal erosion risks factors assessment is then constructed. The final hierarchical structure is achieved, as shown in Fig. 2.

It consists of two main factors which are "geo-environmental hazard" and "socio-economic vulnerability", each of which is divided into seven and five sub-factors respectively.

This study area is at Setiu Wetlands coastal areas in Terengganu, Malaysia. This study is investigated the coastal erosion risks factors around the Setiu Wetlands. In assessing the coastal erosion risks factors, the main factors' weights and sub-factors' weights must be significantly considered. To acquire that, a group of three decision makers consists of a senior supervisor and two experienced coastal engineer, is formed. They are asked to provide judgments via questionnaires and along with our guidance. They need to compare two main factors and twelve sub-factors in the scope of this study.

Pairwise comparison matrices are constructed to analyse the decision makers' judgments and find out the relative importance of one factor over another. The fuzzy pairwise comparison matrix of all three decision makers' judgments when comparing main factors is shown in Table 3.

Table 3. Fuzzy Pairwise Comparison Matrix of factors

	GEH	SEV
GEH	$(1,1,1)$	$(1,3/2,2)$ $(1,1,1)$ $(1/2,2,3/2)$
SEV	$(1/2,2/3,1)$ $(1,1,1)$ $(1,1,1)$ $(2/3,1/2,2)$	

GEH and SEV are the codes stand for Geo-environmental Hazard and Socio-economic Vulnerability respectively. By employing Eq. (19), the group pairwise comparison matrices can be obtained. The group pairwise comparison matrix acquired when making pairwise comparisons of the main factors with respect to the goal is shown in Table 4.

Table 4. Group Fuzzy Pairwise Comparison Matrix of factors

	GEH	SEV
GEH	$(1,1,1)$	$(0.5,1.14,2)$
SEV	(0.5, 0.87, 2)	$(1,1,1)$

Next, the consistency measure is calculated using cosine consistency index. In order to measure the consistency, the triangular fuzzy numbers must be converted into crisp numbers. Decision makers can determine the value of α-cut subjectivity, depending on environmental uncertainty. In this study, α = 0.5 was used to denote that environmental uncertainty is steady, and λ = 0.5 expresses that the attitude is fair. Taking pairwise comparison matrix of the factors in Table 4 as an example, when α and λ = 0.5, the defuzzification was performed as follows:

$$l_{12}^{0.5} = (1.14 - 0.5) \times 0.5 + 0.5$$
$$l_{12}^{0.5} = 0.82$$
$$u_{12}^{0.5} = 2 - (2 - 1.14) \times 0.5$$
$$u_{12}^{0.5} = 1.57$$
$$\left(a_{12}^{0.5}\right)^{0.5} = \left[0.5 \cdot 0.82 + (1 - 0.5) \cdot 1.57\right]$$
$$\left(a_{12}^{0.5}\right)^{0.5} = 1.197$$

Table 5 shows the defuzzified matrix of main factors.

Table 5. Defuzzified matrix

	GEH	**SEV**
GEH	1	1.19
SEV	0.84	1

Normalize the matrix of Table 5 to a transformation matrix using (15b). Table 6 presents the normalized matrix for main factors.

Table 6. Normalize matrix of main factors

	GEH	**SEV**
GEH	0.793	0.80
SEV	0.61	0.60

Then the optimal objective function value are calculated by (15a), we get

$$C^* = 2.0$$

Then

$$CCI = C^*/n$$
$$= 2/2$$
$$= 1 \text{ or } 100\%$$

The CCI value is 1, shows the perfect consistency. Hence, the pairwise comparison matrix of main factors is acceptable. Then, the local weights evaluation can be preceded by Chang's extend analysis of Fuzzy AHP. Local weights of the main factors can be calculated as steps below. First of all, the synthetic extent values of two main factors are determined as follows.

$$S_{GEH} = (1.5, 2.14, 3) \otimes \left(\frac{1}{6}, \frac{1}{4.01}, \frac{1}{3}\right)$$
$$= (0.25, 0.534, 1)$$
$$S_{SEV} = (1.5, 1.87, 3) \otimes \left(\frac{1}{6}, \frac{1}{4.01}, \frac{1}{3}\right)$$
$$= (0.25, 0.466, 1)$$

Then, the minimum degree possibility $d'(A_i)$ of $V(S_i \geq S_k)$ for $k = 1, 2, \ldots, n; k \neq n$ were calculated.

$$d'(GEH) = \min V(S_{GEH} \geq S_{SEV})$$
$$= \min(1)$$
$$= 1$$
$$d'(SEV) = \min V(S_{SEV} \geq S_{GEH})$$
$$= \min(0.92)$$
$$= 0.92$$

Hence, we obtain weights $\overline{W} = (1, 0.92)^T$ and the normalized weights are $W = (0.52, 0.48)^T$. The final weights of the two main factors, GEH and SEV are 0.52 and 0.48 respectively.

For numerical example purposes, the calculations of relative weights for sub-factors will be discussed below in similar way as discussed above. Table 7 shows the group fuzzy comparison matrix of Geo-environmental Hazard (GEH) sub factors. SE, E, SS, TR, RSL, WH and SD stand for shoreline evolution, erodibility, storm surge, tidal range, relative sea level, wave height and sediment discharge respectively.

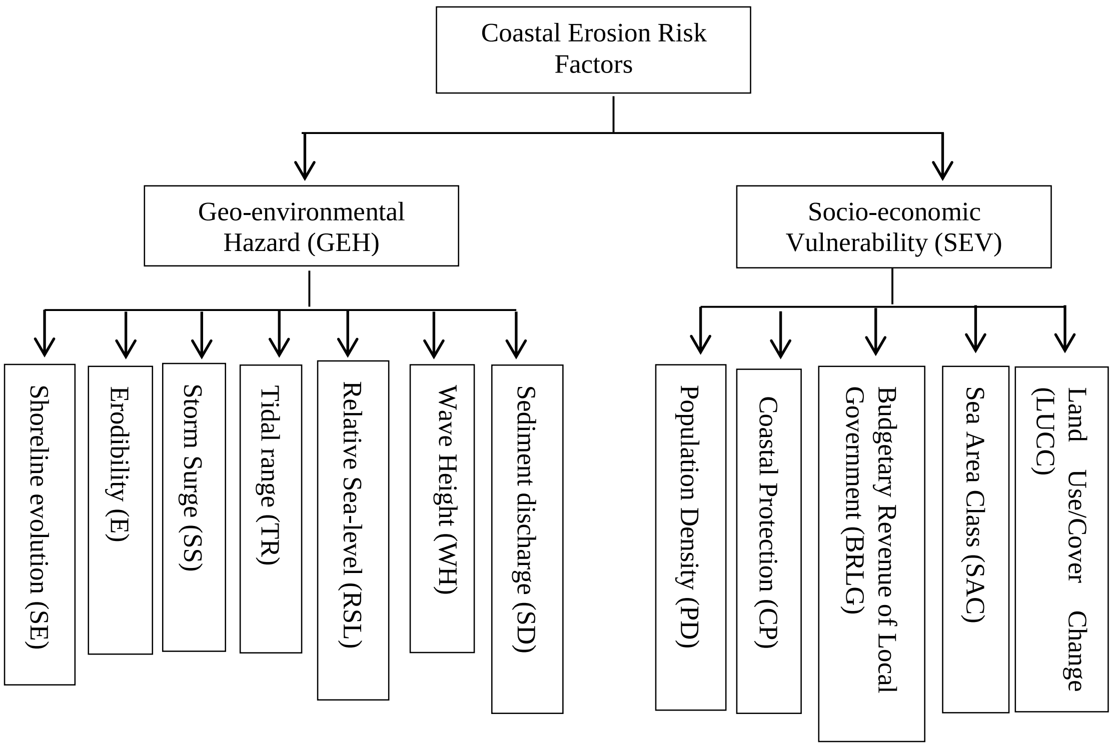

Figure 2. The hierarchy tree

Table 7. Group Fuzzy Comparison Matrix of GEH sub-factors

	SE	E	SS	TR	SD	RSL	WH
SE	$(1,1,1)$	$(0.50,0.76,2.00)$	$(1,1.96,3)$	$(1,1.82,2.50)$	$(1.5,2.62,3.50)$	$(1,1.96,3)$	$(0.50,1.31,2.50)$
E	$(0.50,1.31,2)$	$(1,1,1)$	$(1.5,2.32,3)$	$(1.5,2,2.5)$	$(2.5,3,3.5)$	$(1.5,2.32,3)$	$(1,1.82,2.5)$
SS	$(0.33,0.51,1)$	$(0.33,0.43,0.67)$	$(1,1,1)$	$(0.5,0.76,2)$	$(1,1.96,3)$	$(1,1.96,3)$	$(0.4,0.61,1)$
TR	$(0.4,0.55,1)$	$(0.4,0.5,0.67)$	$(0.5,1.31,2)$	$(1,1,1)$	$(1,2.11,3)$	$(1,1.82,2.5)$	$(0.4,0.61,1)$
SD	$(0.29,0.38,0.67)$	$(0.29,0.33,0.4)$	$(0.33,0.51,1)$	$(0.33,0.47,1)$	$(1,1,1)$	$(0.5,0.76,2)$	$(0.29,0.41,0.67)$
RSL	$(0.33,0.51,1)$	$(0.33,0.43,0.67)$	$(0.33,0.51,1)$	$(0.4,0.55,1)$	$(0.5,1.31,2)$	$(1,1,1)$	$(0.4,0.69,2)$
WH	$(0.40,0.76,1)$	$(0.4,0.55,1)$	$(1,1.65,2.5)$	$(1,1.65,2.5)$	$(1.5,2.47,3.5)$	$(0.5,1.44,2.5)$	$(1,1,1)$

The group fuzzy comparison matrix is defuzzified using (17) to obtain a crisp comparison matrix. Table 8 shows the defuzzified matrix of GEH sub-factors.

Table 8. Defuzzified comparison matrix of GEH sub-factors

	SE	E	SS	TR	SD	RSL	WH
SE	1.00	1.01	1.98	1.78	2.56	1.98	1.41
E	0.99	1.00	2.29	2.00	3.00	2.29	1.78
SS	0.51	0.44	1.00	1.01	1.98	1.98	0.65
TR	0.56	0.50	0.99	1.00	2.05	1.78	0.65
SD	0.56	0.33	0.51	0.49	1.00	1.01	0.44
RSL	0.51	0.44	0.51	0.56	0.99	1.00	0.95
WH	0.71	0.56	1.53	1.53	2.27	1.06	1.00

Normalize the matrix of Table 8 to a transformation matrix using (15b). Table 9 shows the normalize matrix of GEH sub-factors.

Table 9. Normalize matrix of GEH sub-factors

	SE	E	SS	TR	SD	RSL	WH
SE	0.52	0.57	0.53	0.51	0.46	0.45	0.49
E	0.52	0.57	0.61	0.58	0.54	0.52	0.63
SS	0.27	0.25	0.27	0.29	0.36	0.45	0.23
TR	0.29	0.29	0.27	0.29	0.37	0.41	0.23
SD	0.29	0.19	0.14	0.14	0.18	0.23	0.15
RSL	0.27	0.25	0.14	0.16	0.18	0.23	0.33
WH	0.37	0.32	0.41	0.44	0.41	0.24	0.35

From Table 9 above, the optimal objective function value are calculated by (15a), then we get

$C^* = 6.922$

Then

$$
\begin{aligned}
CCI &= C^*/n \\
&= 6.922/7 \\
&= 0.988 \text{ or } 98.8\%
\end{aligned}
$$

The obtained CCI is considered as having a consistent fuzzy comparison matrix, therefore the local weights evaluation can be preceded with Chang's extend analysis of Fuzzy AHP. First of all, the synthetic extent values of eight sub-factors are determined as follows.

$$
\begin{aligned}
S_{SE} &= (6.5,11.43,17.5)\otimes\left(\frac{1}{87.24},\frac{1}{58.72},\frac{1}{37.39}\right) \\
&= (0.075,0.195,0.468)
\end{aligned}
$$

$$
\begin{aligned}
S_{E} &= (9.50,13.77,17.50)\otimes\left(\frac{1}{87.24},\frac{1}{58.72},\frac{1}{37.39}\right) \\
&= (0.109,0.235,0.468)
\end{aligned}
$$

$$
\begin{aligned}
S_{SS} &= (4.57,7.23,11.67)\otimes\left(\frac{1}{87.24},\frac{1}{58.72},\frac{1}{37.39}\right) \\
&= (0.052,0.123,0.312)
\end{aligned}
$$

$$S_{TR} = (4.70, 7.89, 11.17) \otimes \left(\frac{1}{87.24}, \frac{1}{58.72}, \frac{1}{37.39}\right)$$
$$= (0.054, 0.134, 0.299)$$
$$S_{SD} = (3.02, 3.87, 6.73) \otimes \left(\frac{1}{87.24}, \frac{1}{58.72}, \frac{1}{37.39}\right)$$
$$= (0.035, 0.066, 0.180)$$
$$S_{RSL} = (3.30, 5.01, 8.67) \otimes \left(\frac{1}{87.24}, \frac{1}{58.72}, \frac{1}{37.39}\right)$$
$$= (0.038, 0.085, 0.232)$$
$$S_{WH} = (5.80, 9.52, 14.00) \otimes \left(\frac{1}{87.24}, \frac{1}{58.72}, \frac{1}{37.39}\right)$$
$$= (0.066, 0.162, 0.374)$$

The degree of possibility of $S_i \geq S_k$ is calculated as follows.

$V(S_{SE} \geq S_E) = 0.9$
$V(S_{SE} \geq S_{SS}) = 1$
$V(S_{SE} \geq S_{TR}) = 1$
$V(S_{SE} \geq S_{SD}) = 1$
$V(S_{SE} \geq S_{RSL}) = 1$
$V(S_{SE} \geq S_{WH}) = 1$

$V(S_E \geq S_{SE}) = 1$
$V(S_E \geq S_{SS}) = 1$
$V(S_E \geq S_{TR}) = 1$
$V(S_E \geq S_{SD}) = 1$
$V(S_E \geq S_{RSL}) = 1$
$V(S_E \geq S_{WH}) = 1$

$V(S_{SS} \geq S_{SE}) = 0.77$
$V(S_{SS} \geq S_E) = 0.65$
$V(S_{SS} \geq S_{TR}) = 0.96$
$V(S_{SS} \geq S_{SD}) = 1$
$V(S_{SS} \geq S_{RSL}) = 1$
$V(S_{SS} \geq S_{WH}) = 0.86$

$V(S_{TR} \geq S_{SE}) = 0.79$
$V(S_{TR} \geq S_E) = 0.65$
$V(S_{TR} \geq S_{SS}) = 1$
$V(S_{TR} \geq S_{SD}) = 1$
$V(S_{TR} \geq S_{RSL}) = 1$
$V(S_{TR} \geq S_{WH}) = 0.89$

$V(S_{SD} \geq S_{SE}) = 0.45$
$V(S_{SD} \geq S_E) = 0.3$
$V(S_{SD} \geq S_{SS}) = 0.69$
$V(S_{SD} \geq S_{TR}) = 0.65$
$V(S_{SD} \geq S_{RSL}) = 0.88$
$V(S_{SD} \geq S_{WH}) = 0.54$

$V(S_{RSL} \geq S_{SE}) = 0.59$
$V(S_{RSL} \geq S_E) = 0.45$
$V(S_{RSL} \geq S_{SS}) = 0.83$
$V(S_{RSL} \geq S_{TR}) = 0.78$
$V(S_{RSL} \geq S_{SD}) = 1$
$V(S_{RSL} \geq S_{WH}) = 0.68$

$V(S_{WH} \geq S_{SE}) = 0.9$
$V(S_{WH} \geq S_E) = 0.79$
$V(S_{WH} \geq S_{SS}) = 1$
$V(S_{WH} \geq S_{TR}) = 1$
$V(S_{WH} \geq S_{SD}) = 1$
$V(S_{WH} \geq S_{SSL}) = 1$

By using formula (11), we obtain

$$d'(SE) = \min V(S_{SE} \geq S_E, S_{SS}, S_{TR}, S_{RSL}, S_{WH}, S_{SD})$$
$$= \min(0.9, 1, 1, 1, 1, 1)$$
$$= 0.9$$
$$d'(E) = \min V(S_E \geq S_{SE}, S_{SS}, S_{TR}, S_{RSL}, S_{WH}, S_{SD})$$
$$= \min(1, 1, 1, 1, 1, 1, 1)$$
$$= 1$$
$$d'(SS) = \min V(S_{SS} \geq S_{SE}, S_E, S_{TR}, S_{RSL}, S_{WH}, S_{SD})$$
$$= \min(0.77, 0.646, 0.96, 1, 1, 0.86)$$
$$= 0.646$$
$$d'(TR) = \min V(S_{TR} \geq S_{SE}, S_E, S_{SS}, S_{RSL}, S_{WH}, S_{SD})$$
$$= \min(0.79, 0.655, 1, 1, 1, 0.89)$$
$$= 0.655$$
$$d'(RSL) = \min V(S_{RD} \geq S_{SE}, S_E, S_{SS}, S_{TR}, S_{WH}, S_{SD})$$
$$= \min(0.451, 0.30, 0.69, 0.65, 0.88, 0.54)$$
$$= 0.30$$
$$d'(WH) = \min V(S_{RD} \geq S_{SE}, S_E, S_{SS}, S_{TR}, S_{RSL}, S_{SD})$$
$$= \min(0.59, 0.452, 0.83, 0.78, 1, 0.68)$$
$$= 0.452$$
$$d'(SD) = \min V(S_{RD} \geq S_{SE}, S_E, S_{SS}, S_{TR}, S_{RSL}, S_{WH})$$
$$= \min(0.9, 0.79, 1, 1, 1, 1)$$
$$= 0.79$$

Therefore we obtain weights

$$\overline{W} = (0.9, 1, 0.646, 0.655, 0.3, 0.452, 0.79)^T$$

and the normalized weights are

$$W = (0.19, 0.211, 0.136, 0.138, 0.063, 0.095, 0.167)^T.$$

For the sub-factors under the Socio-economic Vulnerability (SEV), the analysis of linguistic data is shown below. Table 10 shows the group fuzzy comparison matrix of SEV sub-factors. LUCC, CP, BRLG, SAC, and PD are the codes stand for land use/cover change, coastal protection, budgetary revenue of local government, sea area class and population density respectively.

Table 10. Group Comparison matrix of SEV sub-factors

	PD	CP	BRLG	SAC	LUCC
PD	$(1,1,1)$	$(1,1.65,2.5)$	$(1,1.65,2.5)$	$(0.4,0.69,2)$	$(0.33,0.43,0.67)$
CP	$(0.4,0.61,1)$	$(1,1,1)$	$(0.5,1.31,2)$	$(0.33,0.46,0.67)$	$(0.29,0.35,0.5)$
BRLG	$(0.4,0.61,1)$	$(0.5,0.76,2)$	$(1,1,1)$	$(0.33,0.46,0.67)$	$(0.29,0.33,0.4)$
SAC	$(0.5,1.44,2.5)$	$(1.5,2.15,3)$	$(1.5,2.15,3)$	$(1,1,1)$	$(0.5,0.76,2)$
LUCC	$(1.5,2.32,3)$	$(2,2.82,3.5)$	$(2.5,3,3.5)$	$(0.5,1.31,2)$	$(1,1,1)$

Table 11 shows the crisp matrix of SEV sub-factors.

Table 11. Defuzzified matrix of SEV sub-factors

	PD	**CP**	**BRLG**	**SAC**	**LUCC**
PD	1.00	1.70	1.70	0.95	0.47
CP	0.59	1.00	1.28	0.48	0.37
BRLG	0.59	0.78	1.00	0.48	0.34
SAC	1.06	2.07	2.07	1.00	1.01
LUCC	2.15	2.68	2.96	0.99	1.00

Table 12 is the normalized matrix of SEV sub-factors to a transformation matrix using (15b)

Table 12. Normalized matrix of SEV sub-factors

	PD	**CP**	**BRLG**	**SAC**	**LUCC**
PD	0.37	0.43	0.39	0.52	0.30
CP	0.22	0.25	0.30	0.26	0.24
BRLG	0.22	0.20	0.23	0.26	0.21
SAC	0.39	0.52	0.48	0.55	0.64
LUCC	0.79	0.67	0.69	0.54	0.63

The optimal objective function value are calculated by (15a), then we get

$C^* = 4.95$

Then

$$CCI = C^*/n$$
$$= 4.95/5$$
$$= 0.99 \text{ or } 99\%$$

The cosine consistency test is 99% which is considered as having an acceptable consistency. Then, the Fuzzy AHP extend analysis is applied as follows.

$$S_{PD} = (3.73, 5.42, 8.67) \otimes \left(\frac{1}{42.91}, \frac{1}{30.34}, \frac{1}{21.44} \right)$$
$$= (0.09, 0.18, 0.41)$$

$$S_{CP} = (2.52, 3.37, 4.67) \otimes \left(\frac{1}{42.91}, \frac{1}{30.34}, \frac{1}{21.44} \right)$$
$$= (0.06, 0.11, 0.22)$$

$$S_{BRLG} = (2.69, 3.4, 5.07) \otimes \left(\frac{1}{42.91}, \frac{1}{30.34}, \frac{1}{21.44} \right)$$
$$= (0.06, 0.11, 0.24)$$

$$S_{SAC} = (5.00, 7.68, 11.50) \otimes \left(\frac{1}{42.91}, \frac{1}{30.34}, \frac{1}{21.44} \right)$$
$$= (0.12, 0.25, 0.54)$$

$$S_{LUCC} = (7.50, 10.45, 13.00) \otimes \left(\frac{1}{42.91}, \frac{1}{30.34}, \frac{1}{21.44} \right)$$
$$= (0.17, 0.34, 0.61)$$

The degree of possibility of $S_i \geq S_k$ is calculated as follows.

$V(S_{PD} \geq S_{CP}) = 1$ $\quad V(S_{BRLG} \geq S_{PD}) = 0.69$

$V(S_{PD} \geq S_{BRLG}) = 1$ $\quad V(S_{BRLG} \geq S_{CP}) = 1$

$V(S_{PD} \geq S_{SAC}) = 0.79$ $\quad V(S_{BRLG} \geq S_{SAC}) = 0.46$

$V(S_{PD} \geq S_{LUCC}) = 0.58$ $\quad V(S_{BRLG} \geq S_{LUCC}) = 0.21$

$V(S_{CP} \geq S_{PD}) = 0.66$ $\quad V(S_{SAC} \geq S_{PD}) = 1$

$V(S_{CP} \geq S_{BRLG}) = 0.99$ $\quad V(S_{SAC} \geq S_{CP}) = 1$

$V(S_{CP} \geq S_{SAC}) = 0.42$ $\quad V(S_{SAC} \geq S_{BRLG}) = 1$

$V(S_{CP} \geq S_{LUCC}) = 0.16$ $\quad V(S_{SAC} \geq S_{LUCC}) = 0.80$

$V(S_{LUCC} \geq S_{PD}) = 1$ $\quad V(S_{LUCC} \geq S_{BRLG}) = 1$

$V(S_{LUCC} \geq S_{CP}) = 1$ $\quad V(S_{LUCC} \geq S_{SAC}) = 1$

By using formula (11), we obtain

$$d'(PD) = \min V(S_{PD} \geq S_{CP}, S_{BRLG}, S_{SAC}, S_{LUCC})$$
$$= \min(1, 1, 0.79, 0.58)$$
$$= 0.58$$

$$d'(CP) = \min V(S_{CP} \geq S_{PD}, S_{BRLG}, S_{SAC}, S_{LUCC})$$
$$= \min(0.66, 0.99, 0.42, 0.16)$$
$$= 0.16$$

$$d'(BRLG) = \min V(S_{BRLG} \geq S_{PD}, S_{CP}, S_{SAC}, S_{LUCC})$$
$$= \min(0.67, 1, 0.46, 0.21)$$
$$= 0.21$$

$$d'(SAC) = \min V(S_{SAC} \geq S_{PD}, S_{CP}, S_{BRLG}, S_{LUCC})$$
$$= \min(1, 1, 1, 0.8)$$
$$= 0.8$$

$$d'(LUCC) = \min V(S_{LUCC} \geq S_{PD}, S_{CP}, S_{BRLG}, S_{SAC})$$
$$= \min(1, 1, 1, 1)$$
$$= 1$$

Therefore we obtain weights $\overline{W} = (0.58, 0.16, 0.21, 0.8, 1)^T$ and the normalized weights are

$$W = (0.211, 0.058, 0.076, 0291, 0.364)^T.$$

In the five sub-factors, LUCC, BRLG and PD are the most preferred sub-factors compared with the other sub-factors. Table 13 shows the computed global weights of all sub-factors.

Table 13 shows the computed global weights of all sub-factors

Table 13. Computed global weights of sub-factors

Factor (Layer 1)	Weights	Sub-factor (Layer 2)	Local weights	Global weights	Global rank
Geo-environmental Hazard (GEH)	0.52	Shoreline evolution (SE)	0.190	0.099	5
		Erodibility (E)	0.211	0.110	3
		Storm Surge (SS)	0.136	0.071	8
		Tidal range (TR)	0.138	0.072	7
		Relative Sea-level (RSL)	0.063	0.033	11
		Wave Height (WH)	0.095	0.049	9
		Sediment discharge (SD)	0.167	0.087	6
Socio-economic Vulnerability (SEV)	0.48	Population Density (PD)	0.211	0.101	4
		Coastal Protection (CP)	0.058	0.028	12
		Budgetary Revenue of Local Government (BRLG)	0.076	0.036	10
		Sea Area Class (SAC)	0.291	0.140	2
		Land Use/Cover Change (LUCC)	0.364	0.174	1

5. Discussion

The result of this study shows that the weightage of main factors (GEH and SEV) are quite similar. This means that these two main factors are almost equally important towards coastal erosion. When examining the sub-factors of coastal erosion, it is seen that LUCC (17.4%), SAC (14%) and E (11%) are the top three most preferred sub-factors compared to the other sub-factors. These followed by PD (10.1%), SE (9.9%), SD (8.7%), TR (7.2%) and SS (7.1%) Therefore, these sub-factors are the key risk factors that should be considered in any coastal erosion management, assessment or mitigation plans. The other sub-factors just barely contributed to the coastal erosion; and WH (4.9%), BRLG (3.6%), RSL (3.3%) and CP (2.8%).

The obtained result is reliable since it has passed through the CCI test and in other word; the judgments made by decision makers are consistent during the judgmental elicitation.

6. Conclusion

The cosine consistency index (CCI) has successfully implemented to check the consistency level of pairwise comparison matrices involving fuzzy numbers. The consistency index is easy to compute and able to synchronize with Fuzzy AHP. The obtained CCI value reflects the consistency level of fuzzy pairwise comparison matrix. However, future studies can explore on the threshold for CCI for achieving desirable value which has not been derived yet using relationship between pairwise comparison matrix consistency and CCI.

The focused of this paper which is to introduce CCI into Fuzzy AHP and applied it to find out the relative weights of risk factors of coastal erosion has been achieved. Fuzzy AHP has properly reflected the vagueness associated with human thoughts. The advantage of using a fuzzy approach is to assign the relative importance of attributes using fuzzy numbers instead of precise crisp numbers. Multiple criteria decision making techniques based on Fuzzy AHP helps to choose the best decision- making strategy using a weighting process through pairwise comparisons.

The coastal erosion assessment has become an important study before applying any coastal management plans. The potential factors that contribute towards coastal erosion are the key towards coastal erosion mitigation plans. The results of this research can provide decision makers with a strategic approach for coastal management and selecting the most effective mitigation plans. The government also can carry out a better solution for the coastal erosion problem.

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