Numerical Solution of Hirota-Satsuma Coupled MKdV Equation with Quantic B-Spline Collocation Method

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***Abstract*:** Collocation method using quintic B-splines finite element have been developed for solving numerically the Hirota-Satsuma coupled MKdV equation. Accuracy of the proposed method is shown numerically by calculating conservation laws, L_2 and L_∞ norms on studying of a soliton solution. It is shown that the collocation scheme for solutions of the MKdV equation gives rise to smaller errors and is quite easy to implement. Numerical experiments support these theoretical results.

Keywords: Hirota-Satsuma Coupled MKdV equation, Collocation method, B-spline method, Finite element method.

1. Introduction

Nonlinear coupled partial differential equations are very important in a variety of scientific fields, especially in fluid mechanics, solid state physics, plasma physics, plasma waves, capillary-gravity waves and chemical physics. The effort in finding exact and numerical solutions to a nonlinear equation is important for the understanding of most nonlinear physical phenomena. In this paper, we consider a generalized Hirota-Satsuma coupled MKdV equations which were introduced by Wu et al. In [1], the authors introduced a 4×4 matrix spectral problem with three potentials and proposed a corresponding hierarchy of nonlinear equations. The coupled MKdV system is very complicated and not easy to solve by direct integration method and the homogeneous balance method et al.

The generalized Hirota-Satsuma coupled KdV and coupled MKdV equations have been studied by many authors via different approaches. Recently, Fan [2] has provided a suggestion to construct soliton solutions for these equation by using an extended tanh-function method and symbolic computation. Solitary solutions for various nonlinear wave equations have been investigated using different methods which can only solve special kind of nonlinear problems due to the limitations or shortcomings in the methods. The type of equations we are handing is attracting many researches and a great deal of work has already been done, for example, Jacobi elliptic function method by Yu et al.[3], the projective Riccati equations method by Yong and Zhang [4], the algebraic method by Zayed et al.[5], variational iteration method by He and Wu [6], Adomian decomposition method by Kaya [7] and homotopy perturbation method by Ganji and Rafei [8]. The aim of this study is to establish the finite element B-spline collocation method for solving MKdV equation. The outline of this work is as follows

In section 2, the governing equation is introduced. Section 3 contains the quintic B-spline collocation method to solve the our interest problem. Finally in section 4, to test the ability of the proposed scheme numerical and exact solutions is compared for a test problem.

2. Governing equations

Two typical equations in hierarchy are as follows

A new generalized Hirota-Satsuma coupled KdV equation

$$\begin{aligned} u_t &= \frac{1}{2}u_{xxx} - 3uu_x + 3(vw)_x, \\ v_t &= -v_{xxx} + 3uv_x, \\ w_t &= -w_{xxx} + 3uw_x, \end{aligned} \tag{1}$$

and a new coupled MKdV equation

$$\begin{aligned} u_t &= \frac{1}{2}u_{xxx} - 3u^2u_x + \frac{3}{2}v_{xx} + 3(uv)_x - 3\lambda u_x, \\ v_t &= -v_{xxx} - 3vv_x - 3u_xv_x + 3u^2v_x + 3\lambda v_x, \end{aligned} \tag{2}$$

where the subscripts t and x denote differentiation with respect to t and x, and λ is an arbitrary constant. Equation (1) becomes a generalized KdV equation for $u=0$ and MKdV equation for $v=0$, respectively [2].

Here we consider the Hirota-Satsuma coupled MKdV equation as

$$\begin{aligned} & u_t - \frac{1}{2}u_{xxx} + 3u^2u_x - \frac{3}{2}v_{xx} - 3(uv)_x + 3\lambda u_x = 0, \\ & v_t + v_{xxx} + 3vv_x + 3u_xv_x - 3u^2v_x - 3\lambda v_x = 0, \end{aligned} \tag{3}$$

with the following boundary and initial conditions

$$\begin{aligned} & u(a,t)=\beta_1, u(b,t)=\beta_2, v(a,t)=0, v(b,t)=0, \\ & u_x(a,t)=0, u_x(b,t)=0, v_x(a,t)=0, v_x(b,t)=0; \\ & \qquad t\in(0,T], \\ & u(x,0)=f(x), \quad v(x,0)=-g(x), \quad a\le x\le b, \end{aligned} \tag{4}$$

where λ is positive parameter and subscripts x and t denote differentiation and $f(x)$ and $g(x)$ are localized disturbance inside the considered interval and will be chosen later.

In sequence a numerical scheme based on finite element method is proposed to solve the problem (3)-(4).

3. Quintic B-spline collocation method (QBCM)

Partition the interval $[a,b]$ as $a = x_0 < x_1 < \ldots < x_N = b$, $h = x_m - x_{m-1}$, $m = 1,2,\ldots,N$. At the nodes x_m, the quintic B-splines Q_m, $m = -2,\ldots,N+2$, are defined by formula 5

$$Q_m(x) = \frac{1}{h^5}\begin{cases} (x-x_{m-3})^5, & [x_{m-3}, x_{m-2}] \\ (x-x_{m-3})^5 - 6(x-x_{m-2})^5 & [x_{m-2}, x_{m-1}] \\ (x-x_{m-3})^5 - 6(x-x_{m-2})^5 + 15(x-x_{m-1})^5 & [x_{m-1}, x_m] \\ (x-x_{m-3})^5 - 6(x-x_{m-2})^5 \\ \quad +15(x-x_{m-1})^5 - 20(x-x_m)^5 & [x_m, x_{m+1}] \\ (x-x_{m-3})^5 - 6(x-x_{m-2})^5 + 15(x-x_{m-1})^5 \\ \quad -20(x-x_m)^5 + 15(x-x_{m+1})^5 & [x_{m+1}, x_{m+2}] \\ (x-x_{m-3})^5 - 6(x-x_{m-2})^5 + 15(x-x_{m-1})^5 \\ -20(x-x_m)^5 + 15(x-x_{m+1})^5 + 6(x-x_{m+2})^5 & [x_{m+2}, x_{m+3}] \\ 0 & x < x_{m-3}, x_{m+3} < x \end{cases} \quad (5)$$

For quintic B-splines near end boundaries, it is necessary to introduce 10-additional knots outside the solution domain to provide the support for the quintic B-spline functions, positioned at

$$x_{-5} < x_{-4} < x_{-3} < x_{-2} < x_{-1}$$
$$\textit{and} \quad x_{N+1} < x_{N+2} < x_{N+3} < x_{N+4} < x_{N+5}.$$

The set of quintic B-splines $Q_{-2}, Q_{-1}, \ldots, Q_{N+2}$ forms a basis over the problem domain $[a,b]$, [11].

A numerical solution of (3)-(4) will be derived by using the collocation method based on quintic B-splines. Collocation approximant can be expressed for $u(x,t)$ and $v(x,t)$ in terms of element parameters δ_m and σ_m, respectively, and quadratic B-splines $Q_m(x)$, $m = -2,\ldots,N+2,$ as

$$u_N(x,t) = \sum_{m=-2}^{N+2}\delta_m(t)Q_m(x), v_N(x,t) = \sum_{m=-2}^{N+2}\sigma_m(t)Q_m(x), \quad (6)$$

where Q_m is the quintic B-splines and δ_m and σ_m are time-dependent parameters to be determined from the quintic B-spline collocation form of the Hirota-Satsuma equation.

By using the approximation (6) and quintic B-splines (5), the nodal value u and v and its first and second derivatives u', u'', v' and v'' at the nodes x_m, $m = -2,\ldots,N+2,$ are obtained in terms of the element parameters as

$$u_m = u(x_m) = \delta_{m-2} + 26\delta_{m-1} + 66\delta_m + 26\delta_{m+1} + \delta_{m+2},$$
$$u'_m = u(x_m)' = \frac{5}{h}(\delta_{m+2} + 10\delta_{m+1} - 10\delta_{m-1} - \delta_{m-2}),$$
$$u''_m = u(x_m)'' = \frac{20}{h^2}(\delta_{m+2} + 2\delta_{m+1} - 6\delta_m + 2\delta_{m-1} + \delta_{m-2}),$$
$$v_m = v(x_m) = \sigma_{m-2} + 26\sigma_{m-1} + 66\sigma_m + 26\sigma_{m+1} + \sigma_{m+2},$$
$$v'_m = v(x_m)' = \frac{5}{h}(\sigma_{m+2} + 10\sigma_{m+1} - 10\sigma_{m-1} - \sigma_{m-2}),$$
$$v''_m = v(x_m)'' = \frac{20}{h^2}(\sigma_{m+2} + 2\sigma_{m+1} - 6\sigma_m + 2\sigma_{m-1} + \sigma_{m-2}). \quad (7)$$

Therefore equation (3) leads to a system of $(2N+2)$ algebraic equation in the $(2N+10)$ unknowns. Substituting the collocation approximants (6)-(7) in the system (3) and its evaluation at the knots give a nonlinear system of differential equations as

$$\begin{aligned}&(\delta^0_{m-2} + 26\delta^0_{m-1} + 66\delta^0_m + 26\delta^0_{m+1} + \delta^0_{m+2}) \\ &-\frac{30}{h^3}(\delta_{m+2} - 2\delta_{m+1} + 2\delta_{m-1} - \delta_{m-2}) \\ &+\frac{15}{h^2}d^2_m(\delta_{m+2} + 10\delta_{m+1} - 10\delta_{m-1} - \delta_{m-2}) \\ &-\frac{30}{h^2}(\sigma_{m-2} + 2\sigma_{m-1} - 6\sigma_m + 2\sigma_{m+1} + \sigma_{m+2}) \\ &-\frac{15}{h}v_m(\delta_{m+2} + 10\delta_{m+1} - 10\delta_{m-1} - \delta_{m-2}) \\ &-\frac{15}{h}u_m(\sigma_{m+2} + 10\sigma_{m+1} - 10\sigma_{m-1} - \sigma_{m-2}) \\ &+\frac{15}{h}\lambda(\delta_{m+2} + 10\delta_{m+1} - 10\delta_{m-1} - \delta_{m-2}) = 0,\end{aligned}$$

$$
\begin{aligned}
&(\sigma^0_{m-2}+26\sigma^0_{m-1}+66\sigma^0_m+26\sigma^0_{m+1}+\sigma^0_{m+2})\\
&+\frac{60}{h^3}(\sigma_{m+2}-2\sigma_{m+1}+2\sigma_{m-1}-\sigma_{m-2})\\
&+\frac{15}{h}v'_m(\sigma_{m-2}+26\sigma_{m-1}+66\sigma_m+26\sigma_{m+1}+\sigma_{m+2})\\
&+\frac{75}{h^2}v'_m(\delta_{m+2}+10\delta_{m+1}-10\delta_{m-1}-\delta_{m-2})\\
&-\frac{15}{h}u^2_m(\sigma_{m+2}+10\sigma_{m+1}-10\sigma_{m-1}-\sigma_{m-2})\\
&-\frac{15}{h}\lambda(\sigma_{m+2}+10\sigma_{m+1}-10\sigma_{m-1}-\sigma_{m-2})=0.
\end{aligned}\tag{8}
$$

where ^{0} denotes differentiation with respect to time and

$$d_m=(\delta_{m-2}+26\delta_{m-1}+66\delta_m+26\delta_{m+1}+\delta_{m+2})$$

$$z_m=\frac{5}{h}(\sigma_{m+2}+10\sigma_{m+1}-10\sigma_{m-1}-\sigma_{m-2})$$

are known as nonlinear terms.

Replacing the time derivative of the parameter δ^0 and σ^0 by usual finite difference approximation $\delta^0=\frac{\delta^{n+1}-\delta^n}{\Delta t}$ and $\sigma^0=\frac{\sigma^{n+1}-\sigma^n}{\sigma t}$ and parameter δ and σ by the Crank-Nicolson formulation $\delta=\frac{\delta^{n+1}+\delta^n}{2}$ and $\sigma=\frac{\sigma^{n+1}+\sigma^n}{2}$, gives nonlinear recurrence relationship for time parameters between consecutive times n and $n+1$ as

$$
\begin{aligned}
&\beta_1\delta^{n+1}_{m-2}+\alpha_1\sigma^{n+1}_{m-2}+\beta_2\delta^{n+1}_{m-2}+\alpha_2\sigma^{n+1}_{m-2}+\frac{66}{\Delta t}\delta^{n+1}_m\\
&+\frac{90}{h^2}\sigma^{n+1}_m+\beta_3\delta^{n+1}_{m+1}+\alpha_3\sigma^{n+1}_{m+1}+\beta_4\delta^{n+1}_{m+2}+\alpha_4\sigma^{n+1}_{m+2}\\
&=\beta_4\delta^n_{m-2}-\alpha_1\sigma^n_{m-2}+\beta_3\delta^n_{m-2}-\alpha_2\sigma^n_{m-2}+\frac{66}{\Delta t}\delta^n_m\\
&\quad-\frac{90}{h^2}\sigma^n_m+\beta_2\delta^n_{m+1}-\alpha_3\sigma^n_{m+1}+\beta_1\delta^n_{m+2}-\alpha_4\sigma^n_{m+2},
\end{aligned}\tag{9}
$$

where $m=0,1,\ldots,N$,

$$\beta_1=h^3+15\Delta t-15/2\Delta th^2(d^2_m+\lambda-z_m),$$

$$\beta_2=26h^3-30\Delta t-75\Delta th^2(d^2_m+\lambda-z_m),$$

$$\beta_3=26h^3+30\Delta t+75\Delta th^2(d^2_m+\lambda-z_m),$$

$$\beta_4=h^3-15\Delta t+15/2\Delta th^2(d^2_m+\lambda-z_m),$$
$$\alpha_1=-15\Delta th+15/2\Delta th^2d_m,$$
$$\alpha_2=-30\Delta th+75\Delta th^2d_m,$$
$$\alpha_3=-30\Delta th-75\Delta th^2d_m,$$
$$\alpha_4=-15\Delta th-15/2\Delta th^2d_m.$$

The above system consists of the $(2N+2)$ equations with the $(2N+2)$ unknown parameters. The elimination of parameters δ^{n+1}_{N+2}, $\delta^{n+1}_{N+1},\delta^{n+1}_{-2}$, δ^{n+1}_{-1}, σ^{n+1}_{N+2}, σ^{n+1}_{N+1}, σ^{n+1}_{-2} and σ^{n+1}_{-1} from the system (9), using the boundary conditions $u(a,t)=\beta_1$, $u(b,t)=\beta_2$, $v(a,t)=v(b,t)=0$, $u_x(a,t)=u_x(b,t)=0$ and $v_x(a,t)=v_x(b,t)=0$ enables one to get a solvable $(2N+2)\times(2N+2)$ matrix system. The resulting pentadiagonal matrix system is easily and efficiently solved with a variant of the Thomas algorithms [12].

To initiate element parameters δ^n_m, we must find the initial unknown parameters δ^0_m by means of the following requirements

$$(u_N)_x(a,0)=-\delta^0_{-2}-10\delta^0_{-1}+10\delta^0_1+\delta^0_2=0,$$

$$(u_N)_{xx}(a,0)=\delta^0_{-2}+2\delta^0_{-1}-6\delta^0_0+2\delta^0_1+\delta^0_2=0,$$

$$(u_N)(x_m,0)=\delta^0_{m-2}+26\delta^0_{m-1}+66\delta^0_m+2\delta^0_{m+1}+\delta^0_{m+2}=u(x_m,0),$$

$$(u_N)_{xx}(b,0)=\delta^0_{N+2}+10\delta^0_{N+1}-10\delta^0_{N-1}-\delta^0_{N-2}=0,$$

$$(u_N)_{xx}(b,0)=\delta^0_{N+2}+2\delta^0_{N+1}-6\delta^0_N+2\delta^0_{N-1}+\delta^0_{N-2}=0.$$

$$(v_N)_x(a,0)=-\sigma^0_{-2}-10\sigma^0_{-1}+10\sigma^0_1+\sigma^0_2=0,$$

$$(v_N)_{xx}(a,0)=\sigma^0_{-2}+2\sigma^0_{-1}-6\sigma^0_0+2\sigma^0_1+\sigma^0_2=0,$$

$$(v_N)(x_m,0)=\sigma^0_{m-2}+26\sigma^0_{m-1}+66\sigma^0_m+2\sigma^0_{m+1}+\sigma^0_{m+2}=v(x_m,0),$$

$$(v_N)_{xx}(b,0)=\sigma^0_{N+2}+10\sigma^0_{N+1}-10\sigma^0_{N-1}-\sigma^0_{N-2}=0,$$

$$(v_N)_{xx}(b,0)=\sigma^0_{N+2}+2\sigma^0_{N+1}-6\sigma^0_N+2\sigma^0_{N-1}+\sigma^0_{N-2}=0,$$

$$m=0,\ldots,N.\tag{10}$$

These system are solved by way of the Thomas algorithms. To cope with the nonlinearity in the system (9), parameters δ_i,σ_i, $i=m-2,\ldots,m+2$ in z_m,d_m,k_m is linearized by replacing the nth time parameter $\delta_i^n, \sigma_i^n, i = m-2, \ldots, m+2$ at each timee step. Accuracy of the solutions is increased with the following corrector step applied two or three times at each time step

$$(\delta^*)^{n+1} = \delta^n + \frac{1}{2}(\delta^{n+1} - \delta^n),$$
$$(\sigma^*)^{n+1} = \sigma^n + \frac{1}{2}(\sigma^{n+1} - \sigma^n), \quad (11)$$

where

$$\delta^n = (\delta_{-2}^n, \ldots, \delta_{N+2}^n), \qquad \sigma^n = (\sigma_{-2}^n, \ldots, \sigma_{N+2}^n). \quad (12)$$

The matrix system (10) is solved to get the initial condition parameters. On determining the initial parameters from the system above, calculation of the solutions are iterated using system (9) at successive times. By using the obtained parameters from system (9), nodal values and its derivatives of order 2 can be worked out from equations (7).

4. Numerical experiments

In this section, we will present numerical results of the Hirota-Satsuma equation for two test problems. Accuracy of the proposed numerical method will be measured with discrete L_{2v} and $L_{\infty v}$, L_{2u} and $L_{\infty u}$ error norms

$$\| u - u_N \|_\infty = \max_j | u_j - (u_N^n)_j |,$$
$$\| u - u_N \|_2^2 = h\sum_j^N | (u_j - (u_N^n)_j)^2 |,$$
$$\| v - v_N \|_\infty = \max_j | v_j - (v_N^n)_j |, \quad (13)$$
$$\| v - v_N \|_2^2 = h\sum_j^N | (v_j - (v_N^n)_j)^2 |.$$

Example 1 . For the problem (3)-(4) consider following assumptions

$$u(x,0) = tanh(x),$$
$$v(x,0) = \frac{5}{2} - 2tanh^2(x), \quad (14)$$

and boundary conditions $u(-40,t) = u(60,t) = 0$ and $v(-40,t) = v(60,t) = 0$ together with derivative boundary conditions.

With respect to these conditions the exact solution may be derived as

$$u(x,t) = tanh(x - \frac{5}{2}t),$$
$$v(x,t) = \frac{5}{2} - 2tanh^2(x - \frac{5}{2}t). \quad (15)$$

Now let $\Delta t = 0.1$ and $h = 0.125$, the run of the proposed numerical algorithm is carried up to time $t = 20$ over the problem domain $-40 \le x \le 60$. The maximum root mean square errors and conversation invariant are presented in table 1 and figures 1 and 2.

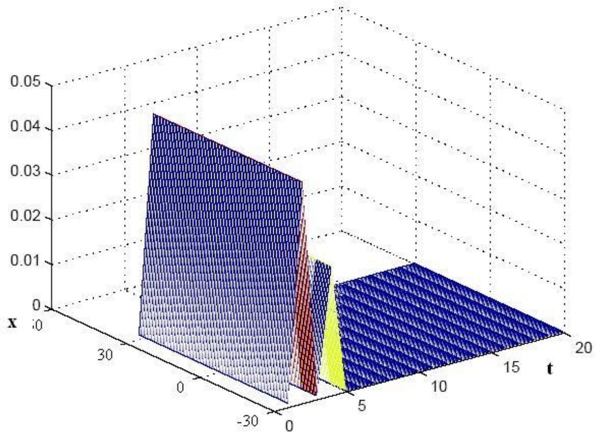

Figure 1. Absolut error between exact and numerical solutions for u .

Example 2 . As another example, the problem (3)-(4) consider following initial and boundary conditions

$$u(x,0) = 1 + tanh(x),$$
$$v(x,0) = 1 + 2tanh(x),$$
$$u(-40,t) = u(60,t) = 0, \quad (16)$$

and $v(-40,t) = v(60,t) = 0$ together with derivative boundary conditions.

Table 1. The comparison between exact and approximate solutions (error norms) when $\Delta t = 0.1$, $h = 0.125$, and $-40 \le x \le 60$

time	$L_{2u} \times 10^3$	$L_{2v} \times 10^3$	$L_{\infty u} \times 10^3$	$L_{\infty v} \times 10^3$
0	0	0	0	0
4	8.459218×10^{-4}	1.343156×10^{-4}	7.422084×10^{-5}	2.380029×10^{-5}
8	7.414878×10^{-4}	1.823145×10^{-4}	1.368505×10^{-4}	2.183528×10^{-4}
12	6.393594×10^{-4}	2.241967×10^{-4}	1.869759×10^{-4}	1.380925×10^{-4}
16	5.703538×10^{-4}	2.596435×10^{-4}	2.215365×10^{-4}	1.53864×10^{-4}
20	5.216854×10^{-4}	2.908314×10^{-4}	2.382659×10^{-4}	1.125429×10^{-4}

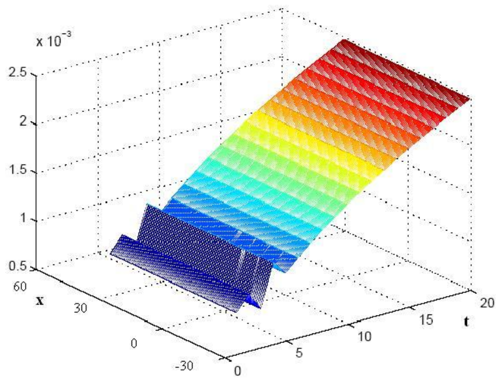

Figure 2. Absolut error between exact and numerical solutions for v.

With respect to these conditions the exact solution can be obtained as

$$u(x,t) = 1 + tanh(x - 2t),$$
$$v(x,t) = 1 + 2tanh(x - 2t). \qquad (17)$$

Similar to the first example we consider $\Delta t = 0.1$ and $h = 0.125$. The run of the algorithms is carried up to time $t = 20$ over the problem domain $-40 \le x \le 60$. The maximum root mean square errors and conversation invariant are presented in table 2 and figures 3 and 4.

5. Conclusion

The main purpose of this paper has been to developed a numerical procedure based on finite element method to solve the MKdV equation. The proposed method have been constructed by using the collocation method with quintic B-splines as interpolation functions. As an advantages of this method one may point out that quintic B-spline functions are handy in writing the approximate solutions in the numerical methods if the partial differential equations involve higher-order derivatives. The proposed method can be used to solve other classes of partial differential equation whit high order derivative if the initial and boundary conditions would be available

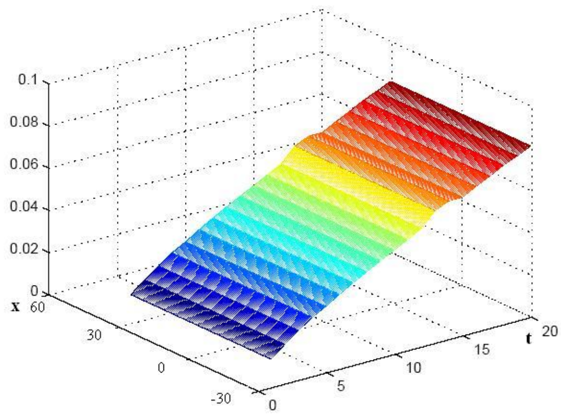

Figure 3. Absolut error between exact and numerical solutions for u.

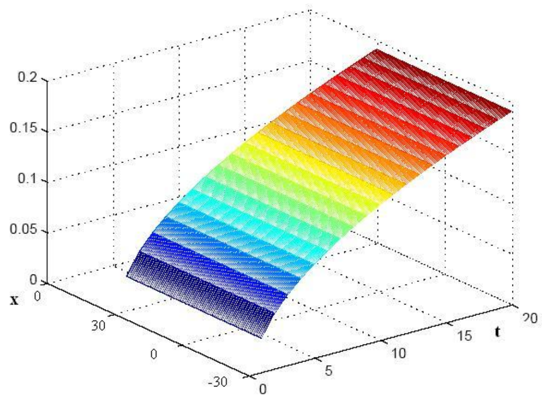

Figure 4. Absolut error between exact and numerical solutions for v.

Table 2. The comparison between exact and approximate solutions (error norms) when $\Delta t = 0.1$, $h = 0.125$, and $-40 \le x \le 60$

time	L_{2u}	L_{2v}	$L_{\infty u}$	$L_{\infty v}$
0	0	0	0	0
4	0.00921616	0.0289541	0.00659199	0.0136491
8	0.01451657	0.0420694	0.00665642	0.0135369
12	0.01930959	0.0519743	0.00654202	0.0138621
16	0.02373622	0.0602728	0.00658223	0.0126491
20	0.02805751	0.0675596	0.00660171	0.0125689

References

[1] Y. T. Wu, X. G. Geng, X. B. Hu and S. M. Zhu, *A generalized Hirota–Satsuma coupled Korteweg–de Vries equation and miura transformations*, Phys. Lett. A. Vol. 255, pp. 259–264, 1999.

[2] E. G. Fan, *Soliton solutions for a generalized Hirota–Satsuma coupled KdV equation and a coupled mKdV equation*, Phys. Lett. A. Vol. 282, pp. 18–22, 2001.

[3] Y. Yu, Q. Wang, H. Zhang, *The extended Jacobi elliptic function method to solve a generalized Hirota–Satsuma coupled KdV nequations*, Chaos Solitons Fractals , Vol. 26, pp. 1415–1421., 2005.

[4] X.L. Yong, H.Q. Zhang, *New exact solutions to the generalized coupled Hirota–Satsuma coupled KdV system*, Chaos Solitons Fractals, Vol. 26, pp. 1105–1110, 2005.

[5] E.M.E. Zayed, H.A. Zedan, K.A. Gepreel, *On the solitary wave solutions for nonlinear Hirota–Satsuma coupled KdV of equations*, Chaos Solitons Fractals, Vol. 22, pp. 285–303, 2004.

[6] J.H. He, X.H. Wu, *Construction of solitary solution and compaction-like solution by variational iteration method*, Chaos Solitons Fractals. Vol. 29, pp. 108–113, 2006.

[7] D. Kaya, *Solitary wave solutions for a generalized Hirota–Satsuma coupled KdV equations*, Appl. Math. Comput., Vol. 147, pp. 69–78, 2004.

[8] D.D. Ganji, M. Rafei, *Solitary wave solutions for a generalized Hirota–Satsuma coupled KdV equations by homotopy perturbation method*, Phys. Lett. A, Vol. 356, pp. 131–137, 2006.

[9] R. Hirota, *J. Satsuma, Phys*, Lett. A , Vol. 85, pp. 407-412, 1981.

[10] J. Satsuma, *R. Hirota*, J. Phys. Soc. Jpn. Vol. 51, pp. 332-343, 1982.

[11] P. M. Prenter, *Splines and variational methods*, John Wiley, Sons, New York, 1975.

[12] V. Rosenberg, *Methods for solution of partial differential equations*, Elsevier, New York, 1969.