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Testing Exponentiality Against Overall Decreasing Life in Laplace Transform Order

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Abstract: This paper is devoted to define a new class of life distribution called overall decreasing life in Laplace transform order ODL_{lt}. A testing hypothesis is constructed to test exponentiality against ODL_{lt}. The critical values of this test are calculated. For Weibull and Gamma alternatives, the power of this test is estimated for different values of the parameter. To evaluate the efficiency of this test, Pitman's asymptotic efficiencies (PAEs) are calculated and compared with some old tests. In case of censored data a testing hypothesis is also discussed. Finally, our proposed test is applied to some real data sets in different areas.

Keywords: **c**lasses of life distributions, overall decreasing life, testing hypothesis, pitman's asymptotic efficiency; testing exponentiality.

1. Introduction

Testing exponentiality versus some classes of life distributions plays an important role in ageing theory. In reliability theory, ageing can be defined as a relationship between the risk failure and age. No ageing means the risk failure is not increase with age (i.e. old is as good as new). Also, proposing new classes of life distributions is very important in theory of reliability. So, many authors proposed a lot of classes and made a testing hypothesis for testing exponentiality versus these proposed classes. The classes of life distributions can be mentioned such as NBU, NBUE, NBUFR, NBAFR, NBURFR, NBARFR, NBRU, NBRUL and EBUL. For more details, to discuss on properties and some possible applications one can refer to Bryson and Siddiqui [1], Deshpande *et al.* [2], Abouammoh and Ahmed [3], Abouammoh *et al.* [4] and Mahmoud *et al.* [5, 6, 7] . Many researchers constructed and discussed some tests for testing exponentiality against some classes of life distributions see Ahmed [8], Abouammoh and Newby [9] , Mahmoud and Abdul Alim [10, 11]. Testing exponentiality against NBU_{mgf} class of life distribution based on Laplace transform has been studied by Atallah *et al.* [12]. Abu youssef *et al.* [13] discussed a test statistics for UBACT based on Laplace transform. Testing NBUCA class of life distributions based on Laplace transform has been studied by Al-Gashgari *et al.* [14].

The rest of this article is organized as follows: Sec. 2 , is devoted to give a brief knowledge renewal classes of life distributions; while testing exponentiality against (ODL_{lt}) class is constructed and discussed in Sec. 3 . Sec. 4 , contains calculations of the PAEs for proposed tests and compared them with some old tests. In Sec. 5 , based on Mont Carlo method, critical values for sample size n = 5(5) 35, 39, 40, 43, 45, 50 are calculated and tabulated. The power estimates for Weibull and Gamma alternatives are found for different values of the parameter in Sec. 6 . The proposed test is handled in case of right-censored data in Sec. 7 . Applying the proposed test for real data sets in different areas is discussed in Sec. 8. Finally, the conclusion is presented in Sec. 9.

2. Renewal Classes of Life Distributions

Let a unit with life time T having a continuous life distribution $F(t)$, is put in operation. At the moment of occurrence failure, it will be replaced by a sequence of mutually independent units. Suppose the units are independent of the first unit and they have the same life distribution $F(t)$. So, In the long run, the renewal survival distribution can be put in the form of

$$\overline{W_F}(t)=\mu_F^{-1}\int_t^{\infty}\overline{F}(u)du, \qquad 0\le t<\infty. \tag{1}$$

where $\mu_F=\mu=\int_0^{\infty}\overline{F}(u)du.$

For more details, see Barlow and Proschan [15] and Abouammoh and Ahmed [3] . In the following, definitions of some classes of life distributions are given.

Definition (2.1)

A random variable T, its life distribution F with $F(0)=0$, or its survival function $\overline{F}$ is said to have increasing (decreasing) failure rate property, denoted by IFR, if

$$\overline{F}(x\mid t)=\frac{\overline{F}(t+x)}{\overline{F}(t)},$$

is decreasing in t, for $x>0,\quad t>0,\ \overline{F}(t)>0$

Definition (2.2)

A life distribution F on $(0,\infty)$, with $F(0^-)=0$ is called overall decreasing life (ODL), if

$$\int_t^{\infty}\overline{W_F}(x)dx\le\mu\overline{W_F}(t),\qquad t\ge 0,$$

this definition is introduced by Sepehrifar *et al.* [16]. It is obvious that ODL contains the IFR class, i.e

$$IFR \Rightarrow ODL\ .$$

New definition of a new renewal class of life distributions is proposed in the following definition.

Definition (2.3)

A life distribution F is said to have the overall decreasing (increasing) life in Laplace transform order, denoted by ODL_{lt} $\left(ODL_{lt}\right)$, if

$$\int_0^\infty \int_t^\infty e^{-st}\overline{W_F}(x)dxdt \le (\ge)\mu\int_0^\infty e^{-st}\overline{W_F}(t)dt, \qquad x,t \ge 0, s \ge 0$$

3. Testing Hypothesis Application

We need to test the following hypothesis

H_0 : F is exponential

H_1 : F is ODL_{lt} and not exponential.

The following lemma is needed.

Lemma (3.1)

If F ε ODL_{lt} class of life distributions, then

$$\frac{1}{s}\mu^2 + \frac{1}{s^2}\mu\zeta(s) \ge \frac{1}{2s}\mu_{(2)} - \frac{1}{s^3}\zeta(s) + \frac{1}{s^3}, s \ge 0, \qquad (2)$$

where $\zeta(s) = Ee^{-sX} = \int_0^\infty e^{-sx}dF(x)$ and $\mu_{(2)}$ is the second moment of F.

Since F is ODL_{lt}, then

$$\int_0^\infty \int_t^\infty e^{-st}\overline{W_F}(x)dxdt \le \mu\int_0^\infty e^{-st}\overline{W_F}(t)dt,, \quad x,t \ge 0, \qquad (3)$$

setting

$$I_1 = \int_0^\infty \int_t^\infty e^{-st}\overline{W_F}(x)dxdt, \qquad (4)$$

I_1 can be rewritten as:

$$I_1 = \frac{1}{s\mu}E\int_0^T (1-e^{-st})(T-t)dt.$$

So,

$$I_1 = \frac{1}{\mu}\{\frac{1}{2s}\mu_{(2)} - \frac{1}{s^2}\mu - \frac{1}{s^3}\zeta(s) + \frac{1}{s^3}\}.$$

Similarly, if we set

$$I_2 = \mu\int_0^\infty e^{-st}\overline{W_F}(t)dt,$$

then

$$I_2 = \frac{1}{s}\mu + \frac{1}{s^2}\zeta(s) - \frac{1}{s^2}, \qquad (5)$$

substituting (4) and (5) into (3), we get

$$\frac{1}{s}\mu^2 + \frac{1}{s^2}\mu\zeta(s) \ge \frac{1}{2s}\mu_{(2)} - \frac{1}{s^3}\zeta(s) + \frac{1}{s^3},$$

which completes the proof.

Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with distribution F. Using Lemma 3.1 and $\gamma(s)$, as a measure of departure from exponentiality, we get

$$\gamma(s) = (\frac{1}{s^2}\mu + \frac{1}{s^3})\zeta(s) + \frac{1}{s}\mu^2 - \frac{1}{2s}\mu_{(2)} - \frac{1}{s^3}. \qquad (6)$$

One can notice that under H_0, $\gamma(s) = 0$, while it is positive under H_1. The empirical estimate $\gamma_n(s)$ of $\gamma(s)$ can be obtained as

$$\gamma_n(s) = \frac{1}{n^2}\sum_{i=1}^n\sum_{j=1}^n[(\frac{1}{s^2}X_i + \frac{1}{s^3})e^{-sX_j} + \frac{1}{s}X_iX_j - \frac{1}{2s}X_i^2 - \frac{1}{s^3}].$$

To make the test invariant, let $\Lambda_n(s) = \frac{\gamma_n(s)}{\overline{X}^2}$, where $\overline{X} = \sum_{i=1}^n \frac{X_i}{n}$ is the sample mean. Then

$$\Lambda_n(s) = \frac{1}{n^2\overline{X}^2}\sum_{i=1}^n\sum_{j=1}^n[(\frac{1}{s^2}X_i + \frac{1}{s^3})e^{-sX_j} + \frac{1}{s}X_iX_j - \frac{1}{2s}X_i^2 - \frac{1}{s^3}]. \qquad (7)$$

It is easy to show that $E[\Lambda_n(s)] = \gamma(s)$.

Consider

$$\phi(X_1, X_2) = (\frac{1}{s^2}X_1 + \frac{1}{s^3})e^{-sX_2} + \frac{1}{s}X_1X_2 - \frac{1}{2s}X_1^2 - \frac{1}{s^3}. \qquad (8)$$

The following theorem summarizes the asymptotic properties of the test statistic $\Lambda_n(s)$.

Theorem (3.1)

As $n \to \infty, [\Lambda_n(s) - \gamma(s)]$ is asymptotically normal with mean 0 and variance $\sigma^2(s)?n$, where

$$\sigma^2(s) = Var\begin{bmatrix}(\frac{1}{s^2}X + \frac{1}{s^3})E\left(e^{-sx}\right) - \frac{1}{2s}X^2 + \frac{2}{s}\mu X \\ + \frac{1}{s^2}\mu e^{-sX} + \frac{1}{s^3}e^{-sX} - \frac{1}{2s}\mu_{(2)} - \frac{2}{s^3}\end{bmatrix}. \qquad (9)$$

Under H_0, the variance can be put in the following form

$$\sigma_0^2(s) = \frac{2s+5}{(s+1)^2(2s+1)}. \qquad (10)$$

Proof:

Making use of the standard U-statistic theory, see Lee [17], yields

$$\sigma^2 = V\{E[\phi(X_1, X_2) \mid X_1] + E[\phi(X_1, X_2) \mid X_2]\}. \qquad (11)$$

Using (8), we can find $E(\phi(X_1, X_2) \mid X_1)$ and $E(\phi(X_1, X_2) \mid X_2)$ as follows

$$E(\varphi(X_1,X_2)\mid X_1)=(\frac{1}{s^2}X+\frac{1}{s^3})\int_0^\infty e^{-sx}dF(x)$$
$$+\frac{1}{s}X\int_0^\infty xdF(x)-\frac{1}{2s}X^2-\frac{1}{s^3},$$

and

$$E(\varphi(X_1,X_2)\mid X_2)=\frac{1}{s^2}e^{-sX}\int_0^\infty xdF(x)+\frac{1}{s^3}e^{-sX}$$
$$+\frac{1}{s}X\int_0^\infty xdF(x)-\frac{1}{2s}\int_0^\infty x^2dF(x)-\frac{1}{s^3},$$

therefore

$$\sigma^2(s)=Var\begin{bmatrix}(\frac{1}{s^2}X+\frac{1}{s^3})E\left(e^{-sx}\right)-\frac{1}{2s}X^2+\frac{2}{s}\mu X\\ +\frac{1}{s^2}\mu e^{-sX}+\frac{1}{s^3}e^{-sX}-\frac{1}{2s}\mu_{(2)}-\frac{2}{s^3}\end{bmatrix}.$$

Under H_0,

$$\sigma_0^2(s)=\frac{2s+5}{(s+1)^2(2s+1)}.$$

4. The Pitman Asymptotic Efficiency (PAE)

To judge on the quality of this procedure, PAEs are computed and compared with some other tests for the following alternative distributions:

The Weibull distribution: $\bar{F}_1(x)=e^{-x^\theta},x\ge 0,\theta\ge 1.$

The linear failure rate distribution (LFR):

$\bar{F}_2(x)=e^{-x-\frac{\theta}{2}x^2},x\ge 0,\theta\ge 0.$

The Makeham distribution:

$\bar{F}_3(x)=e^{-x-\theta\left(x+e^{-x}-1\right)},x\ge 0,\theta\ge 0.$

Note that for $\theta=1$, $\bar{F}_1$ goes to the exponential distribution and for $\theta=0$, $\bar{F}_2$ and $\bar{F}_3$ reduce to the exponential distribution. The PAE is defined by:

$$PAE\ (\Lambda_n(s))=\frac{1}{\sigma_\circ(s)}\left|\frac{d}{d\theta}\gamma(s)\right|_{\theta\to\theta_\circ}.$$

When $s=0.99$, this leads to:

Table 1. Comparison between the PAE of our test and some other tests

Test	Weibull	LFR	Makeham
Kango [18]	0.132	0.433	0.144
Mugdadi and Ahmad [19]	0.170	0.408	0.039
Abdl-Aziz [20]	0.223	0.535	0.184
Mahmoud and Abdul Alim [21]	0.050	0.217	0.144
Our test $\Lambda_n(0.99)$.	0.855	0.982	0.218

From Table1 it is obvious that $\Lambda_n(0.99)$ is better than other tests based on PAEs.

The Upper Percentile Points

Using Mathematica 8 and based on 10000 generated samples of size $n=5(5)35,39,40,43,45,50$ the upper percentile points of statistic $\Lambda_n(0.99)$ for different significance levels 0.1, 0.05 and 0.01 are calculated.

Table 2. Critical values of the statistic $\Lambda_n(0.99)$.

n	90%	95%	99%
5	0.280328	0.311758	0.362439
10	0.208010	0.232653	0.271631
15	0.178997	0.199551	0.230798
20	0.161579	0.180811	0.212579
25	0.150268	0.167017	0.197323
30	0.139040	0.156182	0.185290
35	0.132204	0.149721	0.176857
39	0.125543	0.142791	0.172281
40	0.124523	0.140941	0.168625
43	0.121426	0.137426	0.165003
45	0.120333	0.136246	0.163191
50	0.114586	0.130244	0.156456

From Table 2, it is obvious that the critical values are decreasing as the sample sizes are increasing and they increase as the confidence levels increase.

Power Estimates of the Test $\Lambda_n(0.99)$

In this section the power of our test $\Lambda_n(0.99)$ will be estimated at $(1-\alpha)\%$ confidence level, $\alpha=0.05$ with suitable parameter values of θ at $n=10$, 20 and 30 with respect to Weibull and Gamma distributions based on 10000 samples.

Table 3. The power estimates of $\Lambda_n(0.99)$.

n	θ	Weibull	Gamma
10	2	0.2229	0.6249
	3	0.4215	0.9116
	4	0.6059	0.9822
20	2	0.7261	0.7491
	3	0.9861	0.9771
	4	0.9998	0.9983
30	2	0.9486	0.8147
	3	1.0000	0.9911
	4	1.0000	0.9997

From Table 3, one can note that our test $\Lambda_n(0.99)$ has good power for all alternatives and it increases when the value of the parameter θ and the samples sizes increase.

5. Testing Hypothesis for Right-Censored Data

A test statistic is proposed to test $H_\circ$ against H_1 in case of randomly right-censored (RR-C) data in many practical experiments, the censored data are the only information available. The right-censored model can be described as follows: Suppose n units put under test and $Y_1, Y_2, ..., Y_n$ denote their true life time which are independent identically distributed (i.i.d) with distribution F. Let $Z_1, Z_2, ..., Z_n$ be i.i.d with distribution G. Y 's and Z 's are assumed to be independent. In the RR-C model, we observe the pairs (V_j, δ_j), $j = 1,...,n$ where $V_j = \min(Y_j, Z_j)$ and

$$\delta_j = \begin{cases} 1, & \text{if} \quad V_j = Y_j \ \text{(j - th observation is uncensored)} \\ 0, & \text{if} \quad V_j = Z_j \ \text{(j - th observation is censored)} \end{cases}$$

Let $V_{(0)} = 0 < V_{(1)} < V_{(2)} < ... < V_{(n)}$ denote the orderd V 's and $\delta_{(j)}$ is δ_j corresponding to $V_{(j)}$. Using the censored data (Z_j , δ_j), $j = 1,...,n.$ Kaplan and Meier [22] proposed the product limit estimator as

$$\overline{F}_n(u) = \prod_{[j \,:\, V(j) \le u]} \{(n-j)/(n-j+1)\}^{\delta_{(j)}}, u \in [0, V_{(n)}]$$

Now, for testing $H_\circ$: $\hat{\phi}_c(s) = 0$ against H_1 : $\hat{\phi}_c(s) > 0.$ In case of RR-C data the measure of departure from exponentiallity can be obtained as

$$\hat{\phi}_c(s) = (\frac{1}{s^2}\mu + \frac{1}{s^3})\zeta(s) + \frac{1}{s}\mu^2 - \frac{1}{2s}\mu_{(2)} - \frac{1}{s^3},$$

where $\zeta(s) = \int_0^\infty e^{-sx} dF(x).$

For computational purpose, $\hat{\phi}_c(s)$ may be rewritten as

$$\hat{\phi}_c(s) = (\frac{1}{s^2}\Omega + \frac{1}{s^3})\eta + \frac{1}{s}\Omega^2 - \frac{1}{2s}\Phi - \frac{1}{s^3},$$

where

$$\Omega = \sum_{k=1}^{n}[\prod_{m=1}^{k-1} C_m^{\delta(m)}\left(V_{(k)} - V_{(k-1)}\right)],$$

$$\eta = \sum_{j=1}^{n} e^{-sV_{(j)}}[\prod_{p=1}^{j-2} C_p^{\delta(p)} - \prod_{p=1}^{j-1} C_p^{\delta(p)}],$$

$$\Phi = 2\sum_{i=1}^{n}[\prod_{v=1}^{i-1} V_{(i)} C_v^{\delta(v)}\left(V_{(i)} - V_{(i-1)}\right)],$$

and

$$dF_n(V_j) = \overline{F}_n(V_{j-1}) - \overline{F}_n(V_j), c_k = [n-k][n-k+1]^{-1}.$$

To make the test invariant, let

$$\hat{\Lambda}_c(s) = \frac{\hat{\phi}_c(s)}{\overline{V}^2}, \text{where} \, \overline{V} = \sum_{i=1}^{n}\frac{V_{(i)}}{n}.$$

Table 4, gives the critical values percentiles of $\hat{\Lambda}_c(s)$ test for sample sizes $n = 10(10)50, 51, 60, 70, 81.$ The Monte Carlo null distribution critical values of $\hat{\Lambda}_c(s)$ at $s = 0.99$ for sample sizes n=10 (10) 50, 51, 60, 70, 81 with 10000 replications are simulated from the standard exponential distribution by using Mathematica 8 program. Table 4 gives the upper percentile points of the statistic $\hat{\Lambda}_c(s).$

Table 4. The upper percentile of $\hat{\Lambda}_c(s)$ with 10000 replications at $s = 0.99$

n	90%	95%	99%
10	0.962514	1.916660	4.873790
20	0.537770	1.052880	2.501940
30	0.348216	0.728082	1.794000
40	0.310070	0.608137	1.365730
50	0.267787	0.533167	1.116680
51	0.246944	0.508315	1.077380
60	0.231696	0.458739	1.001290
70	0.213241	0.437317	0.929840
81	0.201485	0.400363	0.791747

Table 4, shows that the critical values decrease as the sample sizes increase and they increase as the confidence level increases.

6. Applications to Real Data

In this section, we apply our test to some real data sets in both non censored and censored data at 95% confidence level.

Non censored data

Example 1: Consider the real data-set given in Grubbs [23] and have been used in Shapiro [24] , this data set gives the times between arrivals of 25 customers at a facility.

Since $\hat{\Lambda}(0.99) = 0.42932$ and this value is greater than the corresponding critical value in Table 2, then we accept H_1 which states that the data set have ODL_{lt} property and not exponential.

Example 2: Consider the data in Abouammoh et al. [4]. These data represent set of 40 patients suffering from blood cancer (Leukemia) from one ministry of health hospital in Saudi Arabia.

Since $\hat{\Lambda}(0.99) = 0.358232$ and this value is greater than the corresponding critical value in Table 2, then we conclude that this data set have ODL_{lt} property and not exponential.

Example 3: Consider the data in Al-Gashgari *et al*. [14] which represent 39 liver cancers patients taken from Elminia cancer center Ministry of Health -- Egypt, which entered in (1999).

In this case, $\hat{\Lambda}(0.99) = 0.215328$ which is greater than the corresponding critical value in Table 2, then we accept H $_1$ which states that the data set have ODL_{lt} property.

Example 4: Consider the following data set is given in Kotz and Johnson [25] and represents the survival times (in years) after diagnosis of 43 patient with certain kind of leukemia.

Since $\hat{\Lambda}(0.99) = 0.191263$ and this value greater than the corresponding critical value in Table 2. Then we conclude that this data set have ODL_{lt} property and not exponential.

Example 5: We consider the real data set in Keating *et al.* [26] which represent the time between successive failures of air conditioning equipment in an aircraft.

In this case $\hat{\Lambda}(0.99) = 0.128554$ which is less than the critical value of Table 2, then we accept H $_o$ which states that the data set have the exponential property.

Censored data

Example 6: Consider the data in Susarla and Vanryzin [27]. These data represent 46 survival times of patients of melanoma, of them 35 represent the whole life times (non-censored data). The ordered censored observations are:

16	21	44	50	55	67	73
76	80	81	86	93	100	108
114	120	124	125	129	130	132
134	140	147	148	151	152	152
158	181	190	193	194	213	215

Taking into account the whole set of survival data (both censored and uncensored), We get $\hat{\Lambda}_c = -3.13408 \times 10^{287}$ which is less than the critical value of the Table 4. Then, H_1 which states that the set of data have ODL_{lt} property is rejected.

Example 7: Consider the data in Mahmoud *et al.* [28] which represent 51 liver cancers patients taken from Elminia cancer center Ministry of Health -- Egypt, which entered in (1999). Out of these 39 represents non-censored data, and the others represents censored data. The censored data are

30	30	30	30	30	60
150	150	150	150	150	185

Taking into account the whole set of survival data (both censored and uncensored), it was found that $\hat{\Lambda}_c = -8.82363 \times 10^{185}$ which is less than the critical value of the Table 4. Then, it is evident to reject at $\alpha = 0.05$ H_1 which states that the set of data have ODL_{lt} property.

7. Conclusion

The ODL_{lt} class of life distributions is defined. It can be considered as a member of the renewal classes. A testing hypothesis is proposed to test exponentiality against this new class based on censored and non-censored data. The upper percentile points and the power estimates are calculated. The PAEs are computed and it is noticed that the PAEs of our new test are better than some old tests for all used alternatives. Finally, our test is applied on some real data sets to show the usefulness of this test.

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