

# Flight Control System Design Optimization via Genetic Algorithm Based on High-Gain Output Feedback

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**Abstract:** In This paper, a high-gain output feedback control structure is applied in order to be optimized with GA for a MIMO nonlinear model of a flight object. In control structure, a modified GA algorithm for obtaining a suitable measurement matrix in feedback loop is proposed which minimizes the interaction between the outputs. The proposed method has two major advantages in compare to other methods; first of all, the proposed method is independent of system degree or system complexity and secondly, in this method some of unknown high-gain method parameters such as arbitrary diagonal matrix, etc are discarded. Computer simulations are carried out for showing the performance of the designed controller against common high-gain controller.

**Keywords:** multivariable systems; flight control; tracking; high-gain output feedback; Genetic Algorithm.

## 1. Introduction

PID controller has widely used as a classical dynamic controller for SISO system. Various PID parameter tuning methods exist for SISO systems (e.g. Ziegler-Nichols method such as C-H-R method [1], and Kitamori's method [2]. Several researches have been conducted about PID control for MIMO systems which they usually restricted to stable and/or minimum phase system.

Several researchers have been used classical control theory for designing and tuning PID control parameters in MIMO system [3], [4], [5]. New research studies have focused on the modern control theory which is more effective for analysis of MIMO system [6], [7], [8], [9], [10]. Lin et al. and Zheng et al., tried to determine PID parameter matrices by solving LMI after one formulates PID control as static output feedback for the extended system [6], [7] But, their method cannot satisfy desired performance in flight control systems because the system outputs are naturally dynamic.

A method by eigenvalue assignment has been proposed in [8]. In all eigenvalue based systems, for each state, a sensor is required. A complicated system such as aircrafts has a significant number of states that in some of them sensors cannot be used directly and should be replaced with observers.

Shimizu and K. Tamura used dynamic high-gain output feedback but the relative degree of system should be less than or equal to two [10].

In order to soften these limitations, GA has been used to optimize high-gain output feedback unknown parameters. In the high-gain output feedback, the closed loop system exhibits a distinctive asymptotic structure in which there are

slow and fast modes. These properties are derived by using singular perturbation method to block diagonalize the closed loop plant. The slow modes are asymptotically uncontrollable or unobservable. Therefore, the output response is dominated by the fast modes. This leads to track the command input by the output quickly.

The design is dependent upon the first markov parameter that is equal to matrix product  $CB$ . In flight systems,  $CB$  does not have maximal rank and is irregular and due to this property, the PI controller is augmented with an inner-loop that provides extra measurements for control purpose. The design of the control law for tracking system may include the desirable requirement if the outputs be decoupled, which minimizes the interaction between the outputs. In order to achieve this decoupling, each of the component slow and fast transfer functions must be diagonal. It may be possible to make these transfer functions diagonal by selecting the measurement matrix  $M$  but, there is no guarantee that this is practical. Ridegely et al. [11] represented a method for accomplishing the selection of the measurement matrix but this method cannot calculate  $M$  in flight or complicated systems.

In order to soften this problem, a modified GA algorithm for obtaining a suitable  $M$  matrix is proposed, which minimizes the interaction between the outputs. The proposed method has two major advantages in compare with other methods; first of all, the proposed method is independent of system degree or system complexity. Secondly, in this method some of unknown high-gain method parameters such as arbitrary diagonal matrix ( $\Sigma$ ), etc are discarded and designers do not need to estimate them.

## 2. High-gain output feedback control

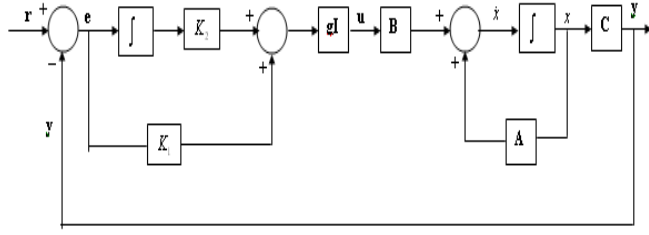
The state and output equations of a MIMO plant are:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t)\end{aligned}\tag{1}$$

Where the dimension of  $A$  is  $n \times n$ ,  $B$  is  $n \times m$ ,  $C$  is  $l \times n$ , and the rank of  $B$  is  $m$ . Suppose the number of controlled outputs  $y(t)$  be equal to number of controls  $u(t)$ . In case of regular plant in which the matrix product  $CB$  has full rank; the High-gain controller implements a proportional plus integral control law, but in case of irregular plants, the PI controller is augmented with an inner-loop that provides

extra measurements through a measurement matrix  $M$  for control purpose. (See Figure 1)

Because of aircraft model irregularity in this section, irregular high-gain control will be illustrated.



**Figure 1.** High-gain Output feedback control for irregular plants.

Using rosenbrok algorithm, the state equation 1 may be transformed to the partitioned form of:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u(t) \quad (2)$$

$$y(t) = [C_1 \quad C_2] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Where  $x_2$  is  $m \times 1$ ,  $B_2$  is  $m \times m$  and has rank  $m$ ,  $C_2$  is  $m \times m$  and has rank  $m$ , and the remaining elements in these equations have appropriate dimensions. This design method requires that the number of controlled outputs  $y(t)$  be equal to the number of controls  $u(t)$  and because of that  $l=m$ . A high-gain controller implements a proportional plus integral (PI) control law represented by

$$u(t) = g\{K_1 e(t) + K_2 z(t)\} \quad (3)$$

Where,  $g$  is a scalar gain. The error vector between the constant command input  $r(t)$  and the output  $y(t)$  is  $e(t) = r(t) - y(t)$ . The integral of the error is the vector  $z(t)$  which satisfies the following equation:

$$z(t) = \int_0^t e(t) dt \Rightarrow \dot{z}(t) = r(t) - y(t) \quad (4)$$

Figure 1 shows a new output described by

$$W(t) = y(t) + M\dot{x}_1 \quad (5)$$

Inserting the values obtained from equation 2 in to equation 5 yields the new output equation, equation 6. By the proper selection of the measurement matrix  $M$ , the matrix  $F_2$  in equation 6 will have full rank.

$$\begin{aligned} w(t) &= [C_1 \quad C_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + M [A_{11} \quad A_{12}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ &= [(C_1 + MA_{11}) \quad (C_2 + MA_{12})] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [F_1 \quad F_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{aligned} \quad (6)$$

The closed-loop tracking system of Figure 1 is represented by equations 3 through 6. Combining these equations, the composite closed-loop state and output equations have the respective forms of:

$$\begin{aligned} \dot{\bar{x}}(t) &= \begin{bmatrix} \bar{A}_1 & \bar{A}_2 \\ \bar{A}_3 & \bar{A}_4 \end{bmatrix} \bar{x}(t) + \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix} r(t) = \bar{A}\bar{x}(t) + \bar{B}r(t) \\ y(t) &= [\bar{C}_1 \quad \bar{C}_2] \bar{x}(t) = \bar{C}\bar{x}(t) \end{aligned} \quad (7)$$

Which the sub matrixes are:

$$\begin{aligned} \bar{A}_1 &= \begin{bmatrix} 0 & -F_1 \\ 0 & A_{11} \end{bmatrix} & \bar{A}_2 &= \begin{bmatrix} -F_2 \\ A_{12} \end{bmatrix} \\ \bar{A}_3 &= [gB_2K_2 \quad A_{21} - gB_2K_1F_1] \\ \bar{A}_4 &= [A_{22} - gB_2K_1F_2] \\ \bar{B}_1 &= \begin{bmatrix} I_l \\ 0 \end{bmatrix} & \bar{B}_2 &= [gB_2K_1 \quad I] \\ \bar{C}_1 &= [0 \quad C_1] & \bar{C}_2 &= C_2 \end{aligned} \quad (8)$$

By using singular perturbation method, the resulting block diagonalization form is:

$$\begin{aligned} \begin{bmatrix} \dot{x}_s(t) \\ \dot{x}_f(t) \end{bmatrix} &= \begin{bmatrix} A_s & 0 \\ 0 & A_f \end{bmatrix} \begin{bmatrix} x_s(t) \\ x_f(t) \end{bmatrix} + \begin{bmatrix} B_s \\ B_f \end{bmatrix} r(t) \\ y(t) &= [C_s \quad C_f] \begin{bmatrix} x_s(t) \\ x_f(t) \end{bmatrix} \end{aligned} \quad (9)$$

Taking the limit as  $g \rightarrow \infty$  yields the components:

$$\begin{aligned} A_s &= \begin{bmatrix} -K_1^{-1}K_2 & 0 \\ A_{12}F_2^{-1}K_1^{-1}K_2 & A_{11} - A_{12}F_2^{-1}F_1 \end{bmatrix} \\ A_f &= -gB_2K_1F_2 \\ B_s &= \begin{bmatrix} 0 \\ A_{12}F_2^{-1} \end{bmatrix} & B_f &= gB_2K_1 \\ C_s &= [C_2F_2^{-1}K_1^{-1}K_2 \quad C_1 - C_2F_2^{-1}F_1], & C_f &= C_2 \end{aligned} \quad (10)$$

The fast transfer function determined from equation 10:

$$\Gamma_f(\lambda) = C_2F_2^{-1}[\lambda I_l + gF_2B_2K_1]^{-1}gF_2B_2K_1 \quad (11)$$

Thus, coefficient matrix  $K_1$  that make  $\Gamma_f(\lambda)$  diagonal is obtained by choosing diagonal matrix  $\Sigma$  as:

$$F_2B_2K_1 = \Sigma = \begin{bmatrix} \sigma_1 & & 0 \\ & \sigma_2 & \\ 0 & & \ddots \\ & & & \sigma_l \end{bmatrix}, \quad K_1 = (F_2B_2)^{-1}\Sigma \quad (12), (13)$$

In High-gain approach usually  $K_2$  is proportional to  $K_1$  as

$$K_2 = -\lambda_0 K_1 \quad (14)$$

The value of  $\lambda_0$  that products satisfactory performance is determined with simulation of the system response. Equations 5 through 14 show that

- $F_2 = C_2 + MA_{12}$  must be non singular.
- For an ideal system,  $C_2F_2^{-1}$  must be diagonal.

Measurement matrix  $M$  should be selected properly in order to satisfy the above conditions. In the mean time, by increasing system gain, close-loop poles move toward

transmission zeroes and M should be able to stabilize transmission zeroes.

### 3. Genetic algorithm

GA is a methodology in evolutionary computation that is commonly used for selecting properly unknown parameters (such as Measurement matrix M) in order to optimize system properties. The genetic algorithm transforms a population of individual objects, each with an associated fitness value, into a new generation of the population. It is based on Darwinian principle of reproduction and survival of naturally occurring genetic operations such as crossover and mutation. The genetic algorithm attempts to find an optimum (or best) solution to the problem by genetically breeding the population of individuals over a series of generations. It is very simple to implement and solves problems very quickly.

In this work we develop a modified GA to find measurement matrix M for obtaining the best parameters for flight control system.

#### 3.1 General methods

We can divide GA methods by three main operators: selection, crossover and mutation.

##### 3.1.1 Selection

- Selection initial samples
  - Randomly create initial samples in search space.
  - Create sequential initial samples in search space.
- Selection scheme in algorithm
  - The probability to choose a certain sample is proportional to its fitness. Algorithm at last is permit to select N/2 samples from N initial samples<sup>1</sup>.
  - Algorithm chooses N/2 samples with better fitness and discards other samples<sup>2</sup>.
  - The probability to choose a certain sample is proportional to its fitness but if the sample with best fitness discards, algorithm replaces this sample with one of selected samples and discards it.

##### 3.1.2 Cross over

- One-point crossover: two strings cut at a randomly chosen position and swapping the two tails. One-point crossover is a simple method for GAs.
- N-point crossover: Instead of only one, N breaking points are chosen randomly. Every second section is swapped.
- Segmented crossover: Similar to N-point crossover with the difference that the number of breaking points can vary.
- Uniform crossover: For each position, it is decided randomly if the positions are swapped.
- Shuffle crossover: First a randomly chosen permutation is applied to the two parents and then N-point crossover is applied to the shuffled parents.

##### 3.1.3 Mutation

- Inversion of single bits: With probability  $P_{mute}$ , one randomly chosen bit is negated.
- Bitwise inversion: The whole string is inverted bit by bit with probability  $P_{mute}$
- Random selection: With probability  $P_{mute}$ , the string is replaced by a randomly chosen one.

Any combination of these operator types makes a GA method. In practice, a desired GA method rapidly and effectively optimizes complex, highly nonlinear, multidimensional systems. A desired GA method should be faster than other methods and more precise.

Among these operators, defining mutation is more crucial than others because of its uncertain nature. Setting this probability higher than critical value, lead to high answer accuracy. The drawback is increasing the numbers of iterations. If this value is assumed smaller than critical value, answer accuracy will be poor and number of iterations will be low. There is a narrow band for this parameter that guarantee answer accuracy with low iteration. Because of certain nature of other parameters in compare with mutation, they are not as important as mutation.

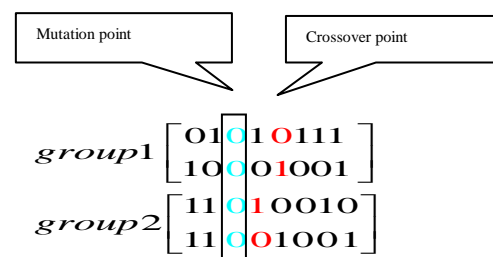
#### 3.2 Proposed method

In this method, mutation operator has been changed for improving GA parameters. Mutation only occurs in positions where bit value of all samples at that position is the same. It is obvious that the mutation in proposed method occurs only in defined bits while general methods apply mutation in all bits. This modified mutation point selection lead to better system performance. Assume one point crossover occur in group1 and group2 at defined positions. As can be seen from Figure 2, after mutation for samples 2 and 4 the bit value in mutation point is negated.

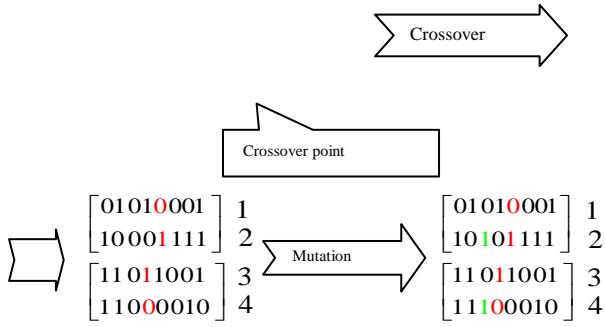
At the next step if child with mutation remain in selection process, the mutation is said to be good and algorithm continues without any change. On the other hand if these children do not remain in selection process, mutation is not appropriate. This indicates increasing the probability of appearing zeroed bit at this location in the final answer. This means that probability that defined bit (at mutation point) be zero at final answer is big.

Facing this conditions lead us to finding a way that decrease the probability of mutation in defined bit. In the above example the first assumed  $P_{mute}$  is 0.5 because the mutation occurred in half of child. As a solution we can assume at next step this probability is 0.25 as a result defined bits in one of four children is negated.

In this method we define different probability of mutation for each position (In Figure 2 eight separate  $P_{mute}$ ). This probability  $P_{mute}$  is similar for all bits comprising the mutation point.


<sup>1</sup> Roulette wheel

<sup>2</sup> Ideal selection



**Figure 2.** Parents and child in an example of proposed algorithm.

In order to evaluate the probability changes in a better way we define  $P_{start}$  instead of  $P_{mute}$ .  $P_{start}$  is the probability of changing bits in mutation point at first mutation which is 0.5 in the above example.  $P_{start}$  is assumed to be equal in all bit positions. Another major parameter in the proposed method is decrease rate which is defined  $De_{rate}$ <sup>3</sup>.  $De_{rate}$  describes the rate of decreasing  $P_{start}$  between two successive mutation steps. In above example  $P_{start}$  is assumed 0.5 and 0.25 for first and second steps, respectively.  $De_{rate}$  is defined by division of these two probabilities which is 0.5. Following this procedure, the third  $P_{mute}$  computed for above example is 0.125 (see 15).

$$\begin{aligned} P_{start} = 0.5 &\rightarrow P_{mute1} = P_{start} \times De_{rate} = 0.25 \rightarrow \\ P_{mute2} &= P_{mute1} \times De_{rate} = 0.125 \end{aligned} \quad (15)$$

It is obvious that  $De_{rate}$  value would be bigger than 1 and should be positive. Setting this parameter close to 1 may result in low convergence rate (very high number of iterations). Setting this parameter to a big value leads to a general mutation system with low  $P_{mute}$ .

#### 4. Controller Design Procedure

High-gain output feedback control is a linear controller. First of all, nonlinear aircraft model is linearized by Jacobian method over accumulation point and then, obtained linear model is used for controller design.

The aircraft model should be included both longitudinal and latitudinal channels. The states used to define longitudinal channel are velocity ( $v$ ), angle of attack ( $\alpha$ ), pitch angle ( $\theta$ ), pitch rate ( $q$ ) and for latitudinal channel are sideslip angle ( $\beta$ ), roll angle ( $\phi$ ), roll rate ( $P$ ), yaw angle ( $\psi$ ), and yaw rate ( $r$ ). The states velocity, pitch angle, yaw angle and sideslip are considered as outputs. Throttle setting ( $\delta_T$ ), elevator deflection ( $\delta_e$ ), aileron ( $\delta_a$ ), and rudder deflection ( $\delta_r$ ) are selected as control inputs. The first and second input- outputs are longitudinal parameters and others are latitudinal which satisfies equality between number of input and output parameters.

The nonlinear state-equations of the flight-system are as followed:

$$\begin{aligned} \dot{w} &= qu - pv + q \cos(\theta) \cos(\phi) + rm\bar{q}s.czt \\ \dot{x}_1 &= \dot{V} = (u\dot{u} + v\dot{v} + w\dot{w})/v_t \\ \dot{x}_2 &= \dot{\alpha} = (u\dot{w} - w\dot{u})/(u^2 + w^2) \\ \dot{x}_3 &= \dot{\beta} = (v\dot{v} - v_x\dot{V}) \cos(\beta) \\ \dot{x}_4 &= \dot{\phi} = p + \frac{\sin(\theta)}{\cos(\theta)} (q \sin(\phi) + r \cos(\phi)) \\ \dot{x}_5 &= \dot{\theta} = q \cos(\phi) - r \sin(\phi) \\ \dot{x}_6 &= \dot{\psi} = (q \sin(\phi) + r \cos(\phi)) / \cos(\theta) \\ \dot{x}_7 &= \dot{p} = (c_2 p + c_1 r + c_4 he)q + \bar{q}sb(c_3.clt + c_4.cnt) \\ \dot{x}_8 &= \dot{q} = (c_5 p - c_7 he)r + c_6(r^2 - p^2) + \bar{q}sc_7.cmt \\ \dot{x}_9 &= \dot{r} = (c_8 p - c_2 r + c_9 he)q + \bar{q}sb(c_4.clt + c_9.cnt) \end{aligned} \quad (16)$$

And in this model the parameters are:

$$\begin{aligned} \bar{q} &= 0.5 \times rho \times v_t^2 \\ rho &= 2.37764 \times 10^{-3} \times (1 - 0.703 \times 10^{-5} h)^{4.14} \\ g &= 32.2, mass = 10136, rm = 1/mass, b = 37.42 \\ s &= 400, he = 0, c_1 = -0.8131, c_2 = -0.0227 \\ c_3 &= 4.3529e-005, c_4 = -5.4603e-007, c_5 = 0.9713 \\ c_6 &= -0.0141, c_7 = 6.6097e-006, c_8 = -0.7570, \\ c_9 &= 5.8911e-006 \end{aligned} \quad (17)$$

The purpose is to design a PI controller with fast and accurate command tracking and to make the sideslip angle approximately zero. This system was in the form of (1) for flight condition of 10000 ft altitude and  $V = 500 ft/sec$ . The respective state matrixes for longitudinal and latitudinal channels after linearization are:

$$A_{long} = \begin{bmatrix} -0.0112 & 48.6809 & -31.7192 & 0.0000 \\ -0.0003 & -0.9866 & -0.0000 & 1.00000 \\ 0 & 0 & 0 & 1.0000 \\ -0.0000 & -2.2665 & 0 & -0.0064 \end{bmatrix}$$

$$B_{long} = \begin{bmatrix} 20.4819 & -0.0000 \\ -0.0005 & -0.0033 \\ 0 & 0 \\ 0 & -0.1774 \end{bmatrix}, \quad C_{long} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad D_{long} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A_{lat} = \begin{bmatrix} -0.2337 & 0.0634 & 0.0120 & -0.999 & 0 \\ 0 & 0 & 1.0000 & 0.0120 & 0 \\ -11.0733 & 0 & -6.0977 & 0.0345 & 0 \\ -0.4310 & 0 & 0.0765 & -0.3721 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

<sup>3</sup> Decrease Rate



$$B_{lat} = \begin{bmatrix} 0.0003 & 0.0011 \\ 0 & 0 \\ -0.4800 & 0.0038 \\ 0.0060 & -0.0409 \\ 0 & 0 \end{bmatrix}, \quad C_{lat} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad D_{lat} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (18)$$

And their eigenvalues are:

$$Eigen\ values_{long} = \{-0.4986 \pm 1.426i, 0.003 \pm 0.097i\} \quad (19)$$

$$Eigen\ values_{lat} = \{0, -1.0321, -6.0714, 0.1990 + 0.05i, 0.1990 - 0.05i\}$$

The eigenvalues show system instability in both of the subsystems. This instability is usual in high maneuverability flight systems.

The effect of interaction should be analyzed in MIMO systems such as aircrafts. The transfer function values for two longitudinal and latitudinal subsystems at  $F=0Hz$  are:

$$G_{long}(0) = \begin{bmatrix} -1.66 & 246.36 \\ 0.64 & -0.2 \end{bmatrix}, \quad G_{lat}(0) = \begin{bmatrix} -0.042 & 0.0651 \\ 0.715 \times 10^{-3} & 0.1 \end{bmatrix} \quad (20)$$

This values show linear model is not diagonal dominance and a central multivariable approach should be used.

Because of system irregularity and complexity, in High-gain output feedback method the measurement matrix is highly important and should be chosen properly. In the mean time, this matrix cannot be obtained by classical methods therefore, in this paper the proposed GA is used for choosing the best measurement matrix from the total set of acceptable measurement matrix.

The parameters used in GA are as follows:

- Variables are the elements of longitudinal and latitudinal measurement matrix.
- Number of variables are 10 (six for latitudinal and 4 for longitudinal matrix).
- All the variable spaces are the same and are equal to  $[-100, 100]$ .
- Number of bits in each variable is selected as 20 so that the total length of each population is 200.
- The number of initial populations is assumed to be 512. In the mean time, the populations that can not satisfy transmission zeros stability or make  $F_2=C_2+MA_{12}$  non full rank are regenerated.
- The number of cross points is assumed to be 9 according to Alavi gharabagh [12].
- According to Alavi gharabagh [12] the parameters  $P_{Start}=0.5$  and  $De_{rate}=1.2$  are applied to guarantee answer accuracy.
- Answer accuracy is a rational factor for breaking computation process.

$$Answer\ Accuracy = \frac{The\ best\ answer - the\ worst\ answer}{The\ best\ answer} \times 100 \quad (21)$$

This value is assumed 0.02%.

- The cost function is the maximum settling time of system outputs which include velocity, pitch, yaw, sideslip angle that should be minimized.

- Based on equations 11, 12 for computing the PI coefficients ( $K_1$  and  $K_2$  matrixes), the diagonal matrix  $\Sigma$  should be determined. In our solution, this matrix is assumed as I (identity matrix), according to equation 12;  $K_2$  is proportional to  $K_1$  and at first is equal to  $K_1$ .

By using GA with above conditions, the best measurement matrixes can be determined as follows:

$$M_{long} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0.15 \end{bmatrix}, \quad M_{lat} = \begin{bmatrix} 0.01 & -49.00 & 3.04 \\ 0.10 & 0.00 & 0.22 \end{bmatrix} \quad (22)$$

Based on equations 11 and 12 the  $K_1$  matrixes are obtained:

$$K_{1long} = \begin{bmatrix} 0.488 & -0.651 \\ 0 & -37.5799 \end{bmatrix}, \quad K_{1lat} = \begin{bmatrix} 0.851 & -0.5830 \\ 0.125 & -11.2213 \end{bmatrix} \quad (23)$$

At the end of simulation and when  $K_1$  is determined, equality of  $K_1$  and  $K_2$  is not necessary and optimum. Therefore, based on some simulations,  $\lambda_{0lat}=0.57$ ,  $\lambda_{0long}=0.57$  are assumed for obtaining  $K_2$  in order to achieve a better performance. With the above assumption

$$K_{2long} = 0.1 \times K_{1long} = \begin{bmatrix} 0.0488 & -0.0651 \\ 0 & -3.75799 \end{bmatrix}$$

$$K_{2lat} = 0.57 \times K_{1lat} = \begin{bmatrix} 0.485 & -0.3323 \\ 0.071 & -6.3396 \end{bmatrix} \quad (24)$$

Finally, by attention to application details such as control efforts the values of gain matrixes are selected which are  $g_{lat}=3I$ ,  $g_{long}=6I$ . The closed-loop eigenvalues for this system are :

$$Eigen\ values_{long} = \{-0.05, -0.05, -0.94, -6.66, -7, -12\}$$

$$Eigen\ values_{lat} = \{-0.05, -0.57, -0.57, -0.25, -2, -4, -4.66\} \quad (25)$$

These eigenvalues emphasize system stability.

## 5. Simulation result

### 5.1 Situations out of accumulation point

In this Simulation, a nonlinear model of a flight object with 6 degrees of freedom is considered. This system outputs should be tracked velocity, pitch and yaw angle commands. In the mean time, sideslip angle must be approximately zero. System is simulated over a wide range of commands around accumulation point.

The close-loop system response for  $v=600$ ,  $\theta=2$ ,  $\psi=10$  is illustrated in Figure 3. The proposed system tracks input commands very good and sideslip angle is approximately zero. In Figure 4, the amplitude of control effort for  $v=600$ ,  $\theta=2$ ,  $\psi=10$  is shown. If the system designed poorly, the amplitude of these signals would be very large and does not work properly for actuators, but in proposed system these values are in the acceptable range.

For illustrating designed controller reliability, a time variant input for pitch and yaw angles is applied to system and the results are shown in Figures 5, 6. the results illustrate the good reliability of proposed system.

In many practical maneuvers, the flight object needs to roll and change yaw angle simultaneously. Figure 7 shows that the proposed system is compatible with this condition.

## 5.2 Comparison between designed PI controller and common PI high-gain controller

In this section, proposed controller is compared with common high-gain controller. In the design of common high-gain controller an acceptable measurement matrix according to the work by [11] is selected. This matrix make  $F_2$  non singular,  $C_2 F_2^{-1}$  diagonal, and transmission zeroes stable. These measurement matrixes are as follows:

$$M_{long} = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad M_{lat} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad (26)$$

Based on equations 11 and 12 the  $K_1$  matrixes are obtained as follows:

$$K_{1long} = \begin{bmatrix} 0.0488 & 0 \\ 0 & -11.2740 \end{bmatrix}, \quad K_{2long} = \begin{bmatrix} 0.0049 & 0 \\ 0 & -1.1274 \end{bmatrix}$$

$$K_{1lat} = \begin{bmatrix} -2.0878 & -0.0562 \\ -0.3063 & -24.4581 \end{bmatrix}, \quad K_{2lat} = \begin{bmatrix} -1.1900 & -0.0320 \\ -0.1746 & -13.9411 \end{bmatrix} \quad (27)$$

From the comparison results, it is obvious that designed controller based on GA is much better than common PI high-gain controller which is shown in Figure 8.

## 6. Conclusion

In this paper a central high-gain output feedback controller for a multivariable flight object optimized by genetic algorithm. Because of linearized plant irregularity, this controller required a suitable measurement matrix. To fulfill this requirement, GA is used. The designed controller was tested on a nonlinear 6 degrees of freedom flight model. All simulation results showed system reliability and stability in practical situations.

In addition, simulation results showed that the time response of designed controller based on GA is much better than common PI high-gain controller. Moreover, the proposed controller has a good performance in a wide range of varieties over accumulation point in compare to other controllers.

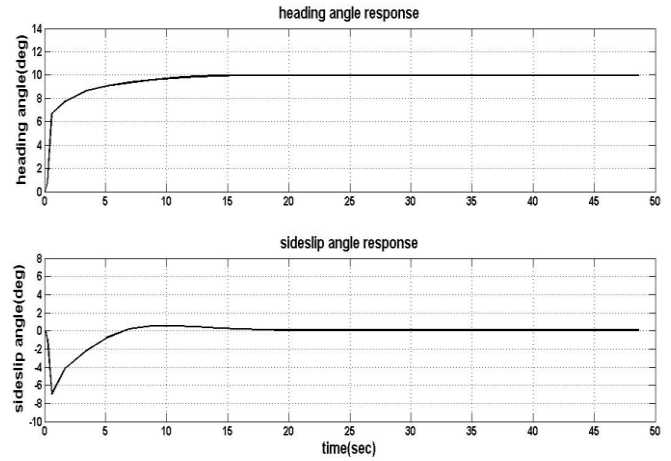
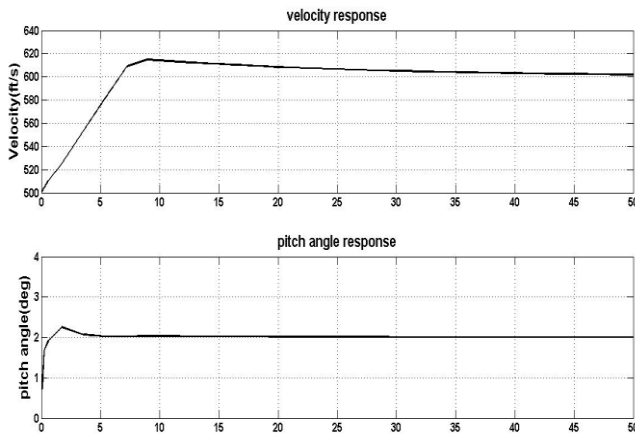


Figure 3. Nonlinear close-loop system response for  $v=600$ ,  $\theta=2$ ,  $\psi=10$

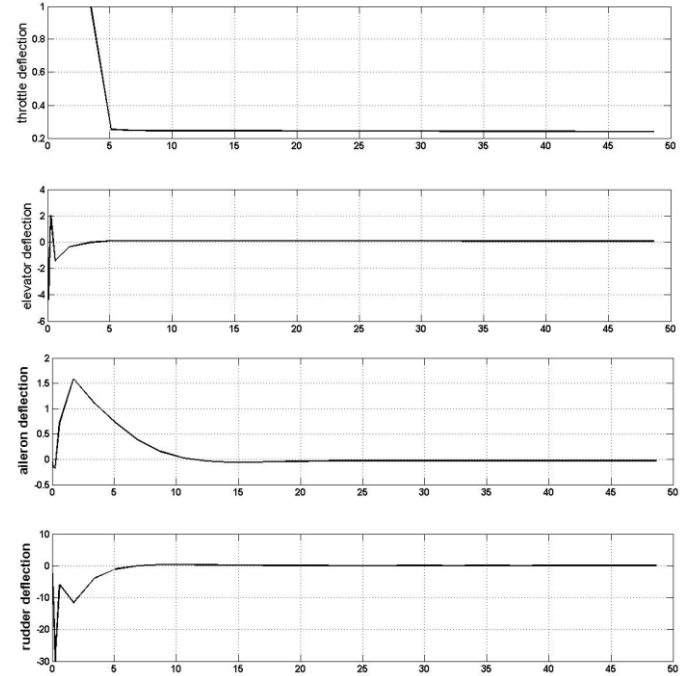


Figure 4. Control efforts for  $v=600$ ,  $\theta=2$ ,  $\psi=10$

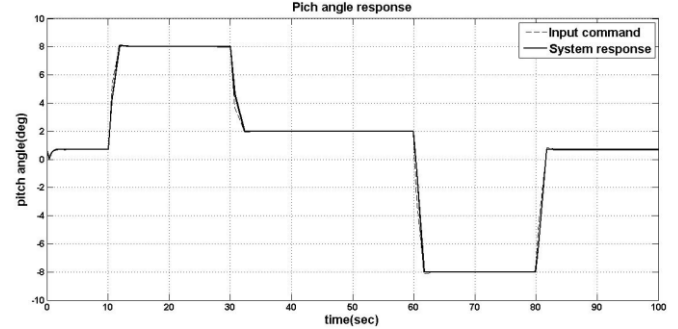
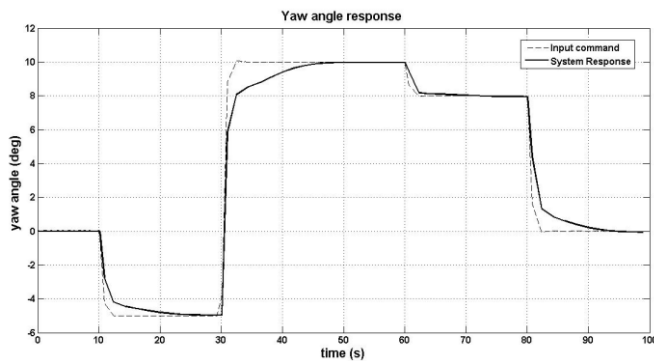
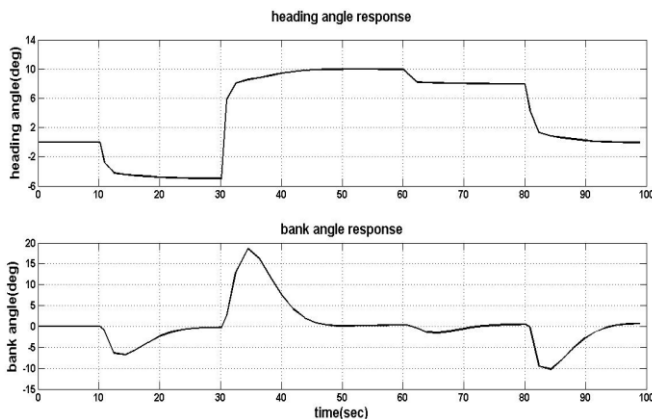


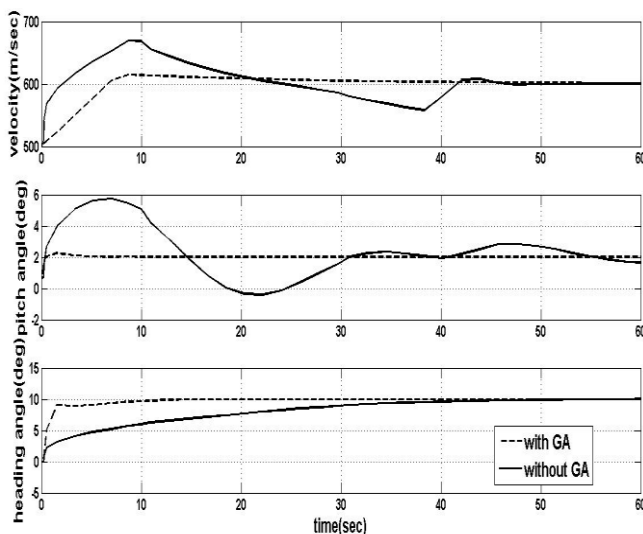
Figure 5. Nonlinear system response for pitch command.



**Figure 6.** Nonlinear system response for yaw command



**Figure 7.** Simultaneous change of roll and yaw angles



**Figure 8.** Designed controller based on GA responses in compare with common PI high-gain controller.

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