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Coupling Tuning Based Impedance Matching for Maximum Wireless Power Transfer Efficiency

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***Abstract*:** In the recent years, a remarkable evolution of the non-radiant wireless power transfer (WPT) system using strong magnetic resonant coupling has been observed, and it has shown more prospects for powering portable devices due to its operating range and efficiency. However, the performance of the system is adversely affected by the coil movement, which is a critical issue for high transfer efficiency. In this paper, a coupling tuning based impedance matching method is proposed to enhance and maximize the transfer efficiency of the system. The matching conditions are derived from power transfer model analysis using equivalent coil circuit model for optimum transfer efficiency. A simulation setup is developed in an electric automatic tool to testify the viability of the proposed technique, and verified against theoretical data. Applying this technique, the extracted S_{21} shows more than 9 dB improvement at both the strong and weak coupling regimes, and results transfer efficiency over 90% at close coil distance without changing the operating frequency. Compared to the conventional fixed coupling and LC circuit based matching technique, the proposed method shows efficiency enhancement about 9% - 20% at different coupling conditions.

***Keywords*:** wireless power transfer, magnetic resonant coupling, impedance matching, coupling, reflected load theory, transmission efficiency**.**

1. Introduction

Nowadays, wireless power transfer (WPT) technology is becoming a top research field for its effective and reliable applicability: from biomedical implants to charging portable electronic devices and high power electric vehicles (EVs) etc. Presently, electromagnetic induction coupling [1-5] and microwave power transfer [6, 7] are reported to be the most popular WPT technologies for consumer applications. But the electromagnetic induction method is limited for short operating range, i.e., typically shorter than 30% of the coil dimension, and microwave power transfer suffers from low efficiency as it uses radiation. For mid-range operation the potential breakthrough of this wireless powering has occurred when the feasibility of the WPT system based on strong magnetic resonant coupling has been proven by MIT [8, 9] theoretically and experimentally.

Based on the principle of magnetic resonant coupling, two resonating coils tuned at the same frequency can effectively exchange energy with greater efficiency at a long operating distance compared to induction method. By using additional intermediate coils as a repeater the power transfer capability of the system can be extended [10, 11]. However, the power transfer capability of the system inversely varies with the distance or the axial misalignment between the coils. Therefore, maximizing efficiency and improving the charging capability over large distance has gained sizeable consideration to make the WPT system more practical. Recent resonant coupled approach [11-13] has proposed multi coil inductive link approach to increase the transmission efficiency for WPT system at large coupling distance by utilizing coupled mode theory (CMT). But CMT only provides accurate model in case of having weak coupled coil with large quality factor and not very convenient to explain the power transfer principle of the magnetic resonant system properly. Recently, a method of automatic adjusting of the source frequency has been suggested [11] to deal with the distance variation and improve transfer efficiency; but this technique often tends the resonant frequency of the system to move out of the usable Industrial, Scientific and Medical (ISM) band of the mid-range WPT system. On the other hand, impedance matching technique using additional matching network on the transmission side [14] satisfies ISM band; but without considering the adaptive matching on the receiving side high transmission efficiency cannot be achieved [15]. However, the additional matching networks increase power losses to the system.

This paper presents a coupling tuning based impedance matching to improve and optimize the transfer efficiency of the 4-coil resonant coupled WPT system during coil separation. To set up the matching condition, the power transfer model of the system is analyzed from an equivalent series-compensated circuit model of the magnetic coupled coils via using reflected load theory (RLT) [16]. Empirical equations for the coupling parameters are established based on impedance matching principle at both transmitting and receiving sides, and the maximum efficiency of the system are calculated as a function of the loaded quality factors and coupling coefficients. This method requires neither manual tuning of source frequency nor additional lossy matching networks, but is developed based on coupling coefficient tuning between the loop-coil and the internal coil combinations of the system. The feasibility of the proposed method is investigated via simulation, and its characteristics are evaluated and compared with some existing methods.

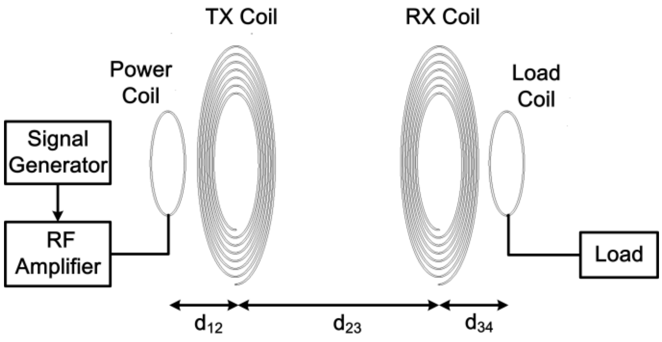


Figure 1. Schematic of the conceptual resonant coupled wireless power transfer system

2. Power Transfer Model of Resonant Based WPT System

2.1 System's Overview

Opposed to the conventional two-coil inductive link system, the resonant coupled system typically depends on four coils which can provide high efficiency even in far field condition. The model of the resonant coupled WPT shown in Figure 1, consists of two loop coils (power and load coils), a transmitting (TX) coil and a receiving (RX) coil. Each loop coil is one of the forms of impedance matching mechanism and is generally made of single turn. Both the TX and RX coils have large number of turns which lead them to act as a high-Q resonator. Certainly, the large Q value of the TX and RX coils can overcome the low coupling between them and helps to maintain high power transmission even when the RX coil moves far away from the TX coil. The key interaction for energy transfer occurs mainly between TX and RX coils and the transfer efficiency of the system practically depends on the distance d_{23} between them. The other two distances between the coil and loop coil combinations are denoted by d_{12} and d_{34}, respectively. Compared to the two-coil counterpart, the four-coil system is less sensitive to the coil distance variation. Therefore, the power transfer model of the resonant coupled WPT needs to be optimized for having maximum efficiency at the operating frequency.

Figure 2 illustrates the series compensated equivalent circuit representation of the four-coil WPT model. This structure makes the compensation capacitance at the power coil independent of the load which helps to maintain resonance during the operation. Both the TX and RX coils are modelled as flat spiral resonators of inductor L_2 and L_3, respectively. R_{P2} and R_{P3} are the parasitic resistance of corresponding coils that causes ohmic losses to the system which dominates the total loss. The geometry of each coil determines their parasitic capacitance represented as C_2 and C_3, respectively. The power coil is modelled as a single loop circular resonator of inductance L_1 and parasitic resistance R_{P1}. Similarly, the load coil and connected load are also modelled as L_4, R_{P4} and R_L, respectively. External capacitors C_1 and C_4 are added to tune the power and load coils at the desired resonant frequency. A simple approximation for the self-inductance of a circular coil can be stated as

$$L_{self} \approx \mu_0\, r\left[\ln\left(\frac{8r}{a}\right)-1.75\right] \tag{1}$$

where, r is the coil radius and a represents the cross-sectional radius of the coil materials. If ω_0 is the resonant frequency, then the parasitic capacitance of each coil can be calculated as

$$C_{self} = \frac{1}{\omega_0{}^2 L_{self}}. \tag{2}$$

When the sinusoidal voltage source V_S is used to excite the power coil connected with an internal resistance of R_S, current I_1 flows through L_1, which results an oscillating magnetic field. The frequency of the AC supply source needs to be equal to the natural resonant frequency of the TX and RX coils so that most of the induced oscillating magnetic field can be cut by the RX and load coils. Otherwise, energy produced by the source would be dissipated in the power coil results in very low efficiency. As all the coils are magnetically connected to each other, they can be characterized by mutual coupling coefficients k_{12}, k_{23} and k_{34}. To make the analysis simple, the strength of cross-coupling between the power and RX coil (k_{13}), the TX and load coil (k_{24}), and the power and load coil (k_{14}) are neglected. Theoretically, the coupling coefficient has a range from 0 to 1 and can be extracted as

$$k_{ij} = \frac{M_{ij}}{\sqrt{L_i L_j}}, \quad (\text{Here, } i, j = 1,\ldots,4) \tag{3}$$

where M_{ij} represents the portion of field linkages between coil i and coil j having N_i and N_j turns, respectively. This can be expressed in terms of mutual inductance as Neumann form as

$$M_{ij} \cong \frac{\pi\mu N_i N_j r_i^2 r_j^2}{2\left[d_{ij}{}^2 + r_j{}^2\right]^{\frac{3}{2}}} \tag{4}$$

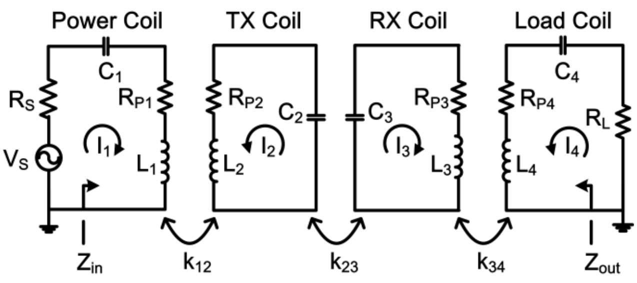


Figure 2. Equivalent circuit model of four coil wireless power transfer system

2.2 Transmission efficiency analysis

From equivalent circuit model shown in Figure 2, it is concluded that the power transfer model of the resonant coupled system can be introduced in a convenient way by adopting circuit based reflected load theory. To ensure the effective power transmission with high Q-factor, the following conditions should be satisfied: $L_2 >> L_1$, $L_3 >> L_4$, $k_{12} >> k_{23}$ and $k_{34} >> k_{23}$. If $\omega_0 = 1/\sqrt{L_i C_i}$ represents the resonant frequency and $Q_i = \omega_0 L_i / R_i$ is the loaded Q-factor of i^{th} coil respectively, the magnitude of the inductive and capacitive reactance of each coil becomes equal at resonance.

$$\begin{aligned} j\omega_0 L_1 + \frac{1}{j\omega_0 C_1} &= j\omega_0 L_4 + \frac{1}{j\omega_0 C_4} = 0 \\ j\omega_0 L_2 + \frac{1}{j\omega_0 C_2} &= j\omega_0 L_3 + \frac{1}{j\omega_0 C_3} = 0 \\ Z_1 = R_S + R_{P1} = R_1, &\quad Z_2 = R_{P2} \\ Z_4 = R_{P4} + R_L = R_4, &\quad Z_3 = R_{P3} \end{aligned} \tag{5}$$

Now, based on reflected load calculation and equation (5), the individual efficiency of each coil can be written as:

$$\eta_4 = \frac{R_L}{R_4}$$

$$\eta_3 = \frac{k_{34}^2 Q_3 Q_4}{1 + k_{34}^2 Q_3 Q_4}$$

$$\eta_2 = \frac{k_{34}^2 Q_3 Q_4}{1 + k_{23}^2 Q_2 Q_3 + k_{34}^2 Q_3 Q_4}$$

$$\eta_1 = \frac{R_1 \left[\dfrac{k_{12}^2 Q_1 Q_2 (1 + k_{34}^2 Q_3 Q_4)}{1 + k_{23}^2 Q_2 Q_3 + k_{34}^2 Q_3 Q_4} \right]}{R_{P1} + R_1 \left[\dfrac{k_{12}^2 Q_1 Q_2 \left(1 + k_{34}^2 Q_3 Q_4\right)}{1 + k_{23}^2 Q_2 Q_3 + k_{34}^2 Q_3 Q_4} \right]}. \tag{6}$$

Therefore, the magnitude of the forward transmission ratio (S_{21}) and the overall power transfer efficiency (η) of the system can be expressed respectively as:

$$|S_{21}| \cong \frac{2 k_{12} k_{23} k_{34} Q_2 Q_3 \sqrt{Q_1 Q_4}}{(1 + k_{12}^2 Q_1 Q_2)(1 + k_{34}^2 Q_3 Q_4) + k_{23}^2 Q_2 Q_3} \tag{7}$$

$$\eta_S = \eta_1 \eta_2 \eta_3 \eta_4 . \tag{8}$$

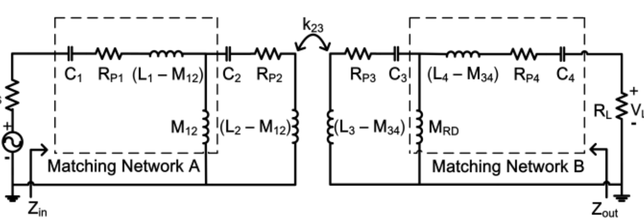


Figure 3. Equivalent circuit schematic of the conceptual impedance matching approach

3. Impedance Matching with Optimum Coupling Tuning

As observed in Figure 2, the input impedance Z_{in} looking into the coupled coils is a function of the mutual coupling between the TX and RX coils, and the output impedance Z_{out} is a function of the transferred power. Therefore, to improve the transmission efficiency, the input and output impedances of the WPT system need to be adequately matched with the complex conjugate of optimal source and load impedances, respectively. This conceptual impedance matching system can be developed by adjusting and tuning of optimum values of k_{12} and k_{34}, respectively, as graphically illustrated in Figure 3. As seen, the matching network A at the transmitting side can be constituted as a T-equivalent circuit by C_1, L_1, C_2 and the coupling coefficient k_{12}. Similarly, C_3, L_4, C_4 and coupling k_{34} together form the T-equivalent matching network B at the receiving side. The magnetizing inductance M_{12} (M_{34}), which is a function of the optimum value of k_{12} (k_{34}), represents the extent of optimum flux linkage between the power and TX coils (RX and load coils). The matching network A acts to match the input impedance Z_{in} at transmitting side with the complex conjugate of the optimum value of R_S, resulting in explicit equations for the optimum coupling k_{12} and Z_{in}, given by

$$k_{12(opt.)} = \sqrt{\frac{1}{Q_1 Q_2} \left(\frac{R_S - R_{P1}}{R_1} \right) \Delta} \tag{9}$$

$$Z_{in(opt.)} = R_{P1} + R_1 \left[\frac{k_{12}^2 Q_1 Q_2 \left(1 + k_{34}^2 Q_3 Q_4\right)}{\Delta^2 + k_{34}^2 Q_3 Q_4} \right] \tag{10}$$

where, $\Delta = \sqrt{1 + k_{23}^2 Q_2 Q_3}$. Similarly, the matching network B also serves to match the output impedance Z_{out} at the receiving side with the complex conjugate of the optimum value of R_L for maximum efficiency. This matching condition acquires corresponding equations for the optimum coupling coefficient k_{34} and optimum load, derived as follows

$$k_{34(opt.)} = \sqrt{\frac{\Delta}{Q_3 Q_4}} \tag{11}$$

$$R_{L(opt.))} = R_{P4} \left[\frac{k_{34}^2 Q_3 Q_4}{\Delta} - 1 \right]. \tag{12}$$

Based on equations (9)-(12), the maximum transfer efficiency can be found from equation (8) as:

$$\eta_{\max} = \frac{\Delta^2 - 1}{[1 + \Delta]^2} \simeq |S_{21}|^2 . \tag{13}$$

From (13), it is shown that the theoretical maximum power transfer efficiency is approximately equivalent to the square of the linear magnitude of S_{21}, i.e., the resonant coupled WPT with the proposed matching technique acts as a two-port network.

4. Validation

The proposed impedance matching approach based on coupling tuning is being verified via simulation using the Advance Design System (ADS) simulator. The symbolic Defined Device (SDD) and Frequency-Defined Device (FDD) models includes equations (4)-(13). This user-defined linear models allow simulation of both large and small signal behaviors for non-linear and high level circuit system. For simplification, each of the TX and RX coils of WPT system are considered as identical flat spiral resonators having approximately 8 turns with 29 cm outer diameter and pitch of 1 cm. With self-capacitance of the spiral loop, the measured self-resonant frequency from the simulation is 12 MHz. On the other hand, the diameter of each loop-coil is 20 cm including single turn only. External capacitors are connected in series with both loop-coil model to set their operating frequency equal to 12 MHz. The cross-sectional radius of the coil wire is assumed to be 2 mm. The extracted lumped parameters for each coil are given in Table I. Also, the extracted value of optimum coupling coefficients k_{12} and k_{34} using equation (9) and (11) are illustrated in Figure 4 as a function of coupling k_{23} over 0.001 to 0.32.

Table 1. Extracted electrical parameters of each individual coil

Parameters	Coil Values			
	Source coil	**TX coil**	**RX coil**	**Load coil**
Diameter (cm)	20	29	29	20
Turn no. (N)	1	8	8	1
Inductance (μH)	0.474	22.22	22.34	0.481
Self-resistance (Ω)	0.213	4.98	6.36	0.2186
Capacitance (pF)	371.11	7.93	7.87	365.7

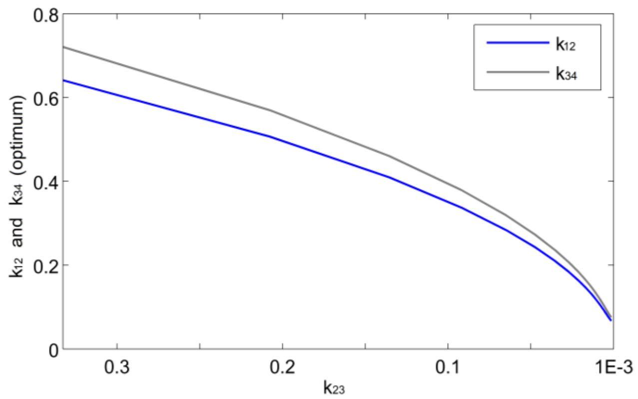
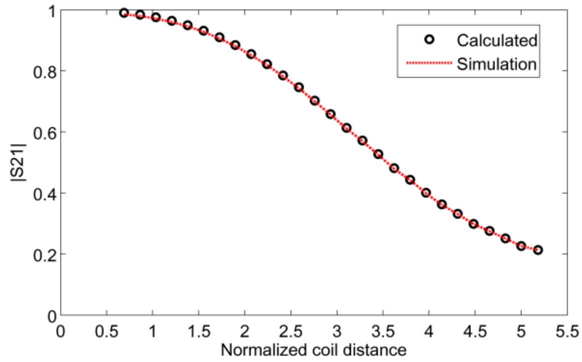
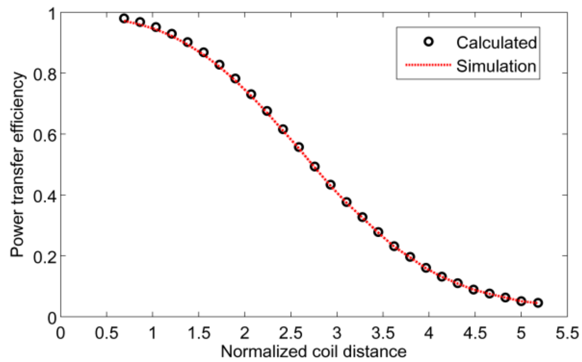


Figure 4. Coupling coefficient (k_{23}) vs coil distance (d_{23})



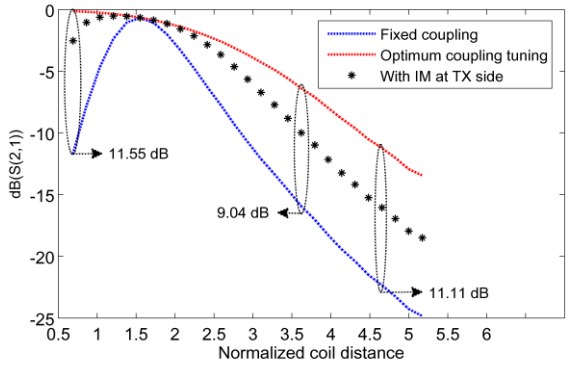
(a)



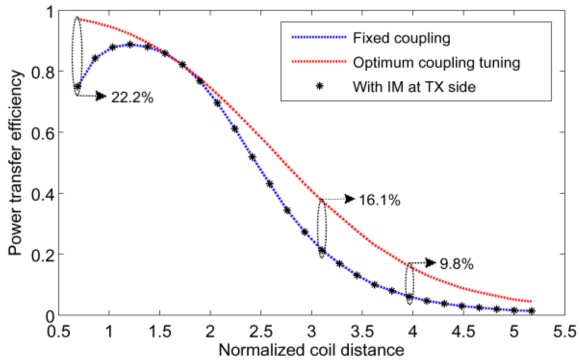
(b)

Figure 5. Theoretical and simulated value of (a) $|S_{21}|$ and (b) power transfer efficiency as a function of normalized coil distance for resonant coupled WPT system with proposed coupling tuning based matching

The resonant coupled WPT can be driven by using a Class E power amplifier which is capable of generating good sine output with low resistance. For simplicity, a sinusoidal voltage source having an internal resistance of 50 Ω is used in simulation instead of Class E amplifier. The impedance value of the load is assumed to be 50 Ω. The operating frequency of the voltage source is set the same as the natural resonant frequency of TX and RX coils. Later, AC sweep simulations are performed to produce frequency-domain solutions much faster than conventional transient time domain solver. The S-parameters toolbox is used to measure the input and output impedances and to adjust the desire couplings respectively for achieving maximum efficiency. The power probe toolbox is used to calculate the maximum transmission efficiency and compare the extracted results with the theoretical value calculated by the derived equations discussed earlier. Figure 5 shows the graphical representation of the theoretical and simulated output for the resonant coupled WPT system, and results in more than 90% transfer efficiency using proposed coupling tuning based matching technique. As seen, the matching technique theoretically generates same the results as simulation except some slight deviations near at close coil distances. This is due to ignoring the parasitic effects of cross-coupling parameters in theoretical analysis. Therefore, for achieving accurate analysis the effect of cross-coupling coefficients are taken account during simulation.



(a)



(b)

Figure 6. Performance comparison of the coupling tuning over the fixed coupling and LC circuit based impedance matching (IM) techniques

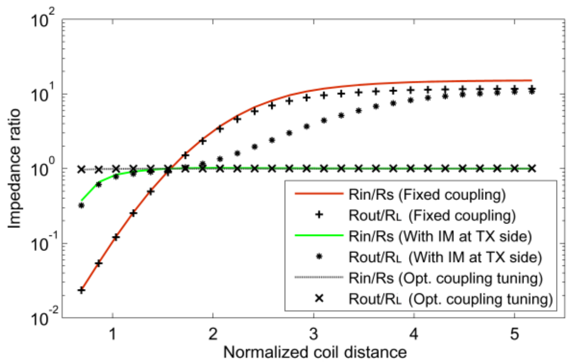


Figure 7. Simulated input and output impedance ratio plot of the resonant coupled WPT where $R_S = R_L = 50\ \Omega$

The comparative study of the performance of the proposed coupling tuning over the conventional fixed coupling and lossy LC circuit based impedance matching (IM) techniques is illustrated in Figure 6. Also, Figure 7 shows the input-output impedance ratio plot of the WPT system which is characterized based on the extracted value of the real component of Z_{in} and Z_{out}. As the overall circuit impedance of the WPT system inversely varies with the square of k_{23}, therefore, a slight mismatch between the source and load impedances significantly affects its power transfer capability. The conventional WPT with fixed coupling (keeping both k_{12} and k_{34} are fixed) technique suffers from poor efficiency at higher value of k_{23} due to the lower value of input and output impedance ratio, i.e., $R_{in}/R_S << 1$ and $R_{out}/R_L << 1$. Likewise for weak k_{23}, the power gain of the system becomes low and both impedance ratios become high (greater than 1) as observed in Figure 6 and Figure 7. The one-sided IM technique reported in [14] using lossy LC circuits can improve S_{21} (Figure 6(a)), but not the efficiency as it considers the matching only at the TX side shown in Figure 7. But without considering the matching at the RX side, the efficiency cannot be maximized. The proposed impedance matching technique based on optimum coupling tuning confirms the adequate matching with source and load impedances (Figure 7), and also results in optimum performance. As seen in Figure 6(b), the coupling tuning method enhances the power transfer efficiency over 90% at close coil distance without shifting the resonant frequency. In fact, S_{21} is intensified more than about 9 dB at both strong and weak coupling regimes illustrated in Figure 6(a). The efficiency improvement is about 9% - 20% at different coupling conditions. Moreover, the conceptual coupling tuning based matching increases S_{21} roughly 1 – 4 dB better than the one-sided IM technique.

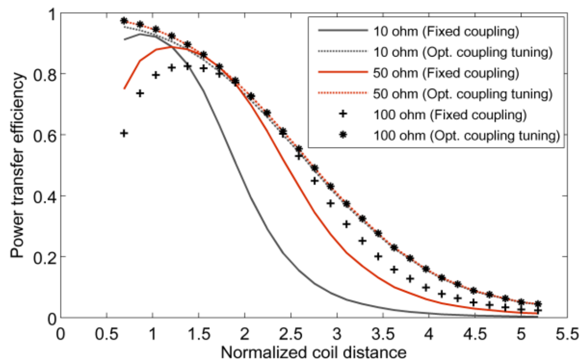


Figure 8. Effect of load variation on the efficiency of resonant coupled WPT system

Figure 8 demonstrates the effect of load variation on the transfer efficiency of resonant coupled WPT using optimum coupling tuning over fixed coupling technique. As seen for close coil proximity, the fixed coupling system with low load resistance provides better efficiency than its higher counterparts. Usually, the small *Q*-factor of the load coil connected with high resistive load shifts and lowers the peak efficiency point compared to the low resistive load. In contrast, the coupling tuning based matching is being less affected by the deviation of load resistance. Also, the proposed method provides approximately the same transfer efficiency for different load resistance without changing the source voltage. The result also confirms the enhancement of the system's operating range with improved efficiency over fixed coupling system.

5. Conclusion

The power transfer efficiency of the resonant coupled WPT is adversely affected by overall circuit impedance variation during coil movements. Also, strong coupling among coils often creates splitting of resonant frequency and lowers the transfer efficiency. This paper presents an impedance matching approach based on coupling tuning at both the transmitting and the receiving sides to improve and maximize the transfer efficiency of a resonant coupled WPT system without changing the operating frequency. Design guidelines for the power transfer model with a conceptual matching technique has been analyzed based on the equivalent circuit model, source and load impedances, and magnetic coupled coils. The effect of coil distance variation is studied through simulation and the results shows improved system performance as well as confirms the adequate input-output impedance matching. Moreover, the proposed technique successfully compensates the effect of load variation, which may lead the industries for implementing future highly efficient cordless charging mechanism for portable electronic devices (e.g. laptops, cell-phone, PDAs, tablets etc.), biomedical implant sensors as well as high power electric vehicles (EVs) in the future.

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