An Improved Combined Shewhart-EWMA Chart based on Double Median Ranked Set Sampling

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***Abstract*:** This study analyzes the performance of a combined Shewhart-EWMA control chart based on double median ranked set sampling (DMRSS), for efficient monitoring of changes in process mean level. The numerical performances of the proposed scheme were evaluated in terms of the average run length (ARL), standard deviation of run length, the average ratio ARL and average extra quadratic loss. Results show that the use of a well-structured sampling method on the combined scheme has greatly enhanced the performance of the scheme in detecting all kinds of shifts in a process. We present a comparison of the proposed scheme with some location charts for monitoring changes in a process. Practical application of the proposed scheme is also demonstrated using real industrial data.

***Keywords*:** average run length, control chart, location chart, ranked set sampling, Shewhart-EWMA**.**

1. Introduction

The Shewhart control chart for location is one of the most widely used statistical process control tools for monitoring the process mean to ensure quality. The scheme is very effective in detecting large process shifts from the target mean level. To detect small changes, Shewhart control charts are effectively complemented by other schemes such as the Exponential Weighted Moving Average (EWMA) control charts. To have a balance against process changes of different sizes, it's often recommended to combine the two control charts probabilistically [1] for efficient monitoring of processes [2].

The combined Shewhart-EWMA (CS-EWMA) control chart combines the key features of the Shewhart and the EWMA charts. The scheme can detect changes in the process that were small enough to escape detection by the Shewhart chart and large enough to escape detection by the EWMA chart. The procedure was suggested by [1] and the need for combined applications of the two schemes was stressed by [3]. In simulations, [4] showed that a CS-EWMA control chart could detect shifts in a process faster than runs rules based Shewhart chart. More recent contributions on the advancements of this combined scheme can be found in [5], [6] and [7] among others.

Ranked set sampling (RSS) technique and some of its modifications have successfully been used to enhance the performance of several control charts in detecting changes in process quality characteristics. For example, see [8-14] to mention but few. RSS based control charts are useful in practical applications where actual measurements of the quality characteristic of interest are quite expensive, time-consuming or destructive, but a set of randomly selected observations could easily be ranked using some subjective judgment or auxiliary measurements, without actual measurement.

The RSS methodology was introduced by [15] in estimating the population mean of pasture and forage yields. It is a well-structured sampling technique that provides significant improvement over the traditional simple random sampling (SRS) in estimating process parameters through additional information resulting from ordered units, which are actually not measured. The technique provides better representative samples than SRS. In this article, new CS-EWMA control chart for location based on double-median variation of RSS (DMRSS) is proposed as an improvement over [7].

The rest of the article is structured as follows. In the next section, we present the CS-EWMA control charts for location based on SRS and RSS methods. Section 3 describes the design structure of the proposed CS-EWMA chart using DMRSS. In Section 4, run length (RL) properties of the proposed charts are given. In Section 5, performance comparisons with some of the existing schemes are provided. Application examples of the proposed scheme using a real industrial data set are given in Section 6. Finally, the conclusions are in Section 7.

2. Brief review of CS-EWMA Control Charts

2.1 Classical CS-EWMA control chart

Let $x_{1j},\ x_{2j},\ x_{3j}, \dots,\ x_{nj}$ for $j = 1, 2, 3, \dots, m$ be independent and identically distributed normal random variables of subgroup size n with mean μ and variance σ^2. Then, the mean $\bar{X}_j = (1/n)\sum_{i=1}^{n} x_{ij}$ of the jth sample is also normally distributed with mean μ and variance σ^2/n. The Shewhart feature of the CS-EWMA control chart is based on the subgroup mean statistic $\bar{X}_j$ and the EWMA component uses

$$Z_j = \lambda\bar{X}_j + (1-\lambda)Z_{j-1} \qquad (1)$$

where λ is the smoothing constant with $0 < \lambda \le 1$. If the process is in-control, the starting value of the statistic is set equal to the target mean, i.e. $Z_0 = \mu_0$. The variance of Z_j is given by

$$\operatorname{var}(Z_j) = (\sigma^2/n)\left(\frac{\lambda}{2-\lambda}\right)[1-(1-\lambda)^{2j}]. \qquad (2)$$

Assume, without loss of generality, that the in-control mean and variance are zero and one, respectively. The CS-EWMA control chart gives an out-of-control signal when either $|\bar{X}_j| > \text{SCL}$ or $|Z_j| > \text{ECL}$ where

$$\text{SCL} = k\left(\frac{\sigma}{\sqrt{n}}\right) \text{ and}$$
$$\text{ECL} = L\left(\frac{\sigma}{\sqrt{n}}\right)\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2j}]} \quad (3)$$

are the Shewhart and EWMA control limits, respectively. The design parameters k and L are chosen based on the choice of the smoothing constant λ to satisfy the required process in-control needs [1].

2.2 CS-EWMA control chart using RSS

The RSS procedure consists selecting n random sets, each of subgroup size n, from a target population. Rank the units within each set by visual inspection or some less expensive method, with respect to a quality characteristic of interest. Select the smallest ranked unit from the first set, the second smallest ranked unit from the second set and so on, until the largest ranked unit is selected from the last set. The cycle may be repeated m times to obtain a sample of nm units of RSS data.

Let $x_{(i:n)j}$ for $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$ denote the ith order statistic from the ith sample of subgroup size n in the jth cycle, then the unbiased estimator for the population mean using RSS is $\bar{X}_{rssj} = (1/n)\sum_{i=1}^{n} x_{(i:n)j}$ with the variance

$$\text{var}(\bar{X}_{rss}) = (1/n^2)\sum_{i=1}^{n}\sigma^2_{(i:n)} \text{ or}$$
$$\text{var}(\bar{X}_{rss}) = \frac{\sigma^2}{n} - \frac{1}{n^2}\sum_{i=1}^{n}\left(\mu_{(i:n)} - \mu\right)^2 \quad (4)$$

where $\mu_{(i:n)}$ and $\sigma^2_{(i:n)}$ are the mean and variance of the ith order statistic, respectively. The value of $\sigma^2_{(i:n)}$ given by

$$\sigma^2_{(i:n)} = \int_{-\infty}^{\infty}\left(x - \mu_{(i:n)}\right)^2 f_{(i:n)}(x)\ dx \quad (5)$$

can be obtained from any known table of order statistics for standard normal distribution, for example, see [16]. $f_{(i:n)}(x)$ is the probability distribution function of $x_{(i:n)j}$ defined as

$$f_{(i:n)}(x) = \frac{n!}{(i-1)!(n-i)!}\int_{-\infty}^{x} F^{i-1}(t)[1-F(t)]^{n-1} f(t)dt. \quad (6)$$

Assume that the process mean is known or can be estimated from the preliminary in-control process data and that the population variance σ^2 is known and constant. The Shewhart and EWMA control limits based on the RSS data are respectively given by [7]

$$\text{SCL}_{rss} = k\sigma_{\bar{X}rss}$$
$$\text{ECL}_{rss} = L\sigma_{\bar{X}rss}\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2j}]} \quad (7)$$

where $\sigma_{\bar{X}rss} = \sqrt{\sigma^2/n - (1/n^2)\sum_{i=1}^{n}\left[\mu_{(i:n)j} - \mu\right]^2}$ is the standard deviation of RSS. The CS-EWMA control chart signals when either the mean $\bar{X}_{rssj}$, exceeds SCL_{rss} or the RSS based EWMA statistic $Z_{(j:n)} = \lambda\bar{X}_{rssj} + (1-\lambda)Z_{(j-1:n)}$ exceeds ECL_{rss}, for details see [7].

3. Proposed CS-EWMA using DMRSS

Statistically, sample median is an outlier-resistant statistic and a robust estimator of location for samples with outlying observations [17]. In RSS setup, the computation of sample median does not require accurate measurement of each individual observation and hence fewer ranking errors. This is called median RSS (MRSS) suggested by [18]. The MRSS procedure for obtaining median ranked set samples of size n is as follows: Select n random samples each of size n from the target population. Rank the units within each set with respect to the variable of interest. If the sample size is odd, select the median value and if the sample size is even, select the two middlemost order-statistics.

To further enhance the performance of the CS-EWMA control chart for monitoring the process mean, we proposed a CS-EWMA chart based on DMRSS. The proposed double median RSS (DMRSS) proposed by [9] can be summarized in the following steps.

Step 1: Select n random sets each of size n^2 units from the parent population.

Step 2: Apply the above MRSS procedure on each set to obtain n median ranked set samples each of size n.

Step 3: Re-apply the MRSS procedure again on the n median ranked set samples obtained in Step 2.

Step 4: The whole process may be repeated m times to obtain nm units of DMRSS samples.

Let $y_{[i:(n+1)/2]j}$ $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$ denotes the median of the ith median ranked set samples if the set size is odd. Also, let $y_{[i:n/2]j}$ and $y_{[i:(n+2)/2]j}$ denotes the $(n/2)$th and $((n+2)/2)$th order statistic of the ith median ranked set samples, respectively, if the set size is even. The estimator of the population mean with its variance based on DMRSS samples for the jth cycle are defined for the odd sample size as

$$\bar{Y}_{dmrssj,o} = (1/n)\sum_{i=1}^{n} y_{(i:(n+2)/2)j}$$
$$var\left(\bar{Y}_{dmrss,o}\right) = (1/n^2)\sum_{i=1}^{n}\sigma^2_{(i:(n+2)/2)} \quad (8)$$

and even sample sizes by

$$\bar{Y}_{dmrssj,e} = \frac{1}{n}\left[\sum_{i=1}^{n/2} y_{(i:n/2)j} + \sum_{i=n/2+1}^{n} y_{(i:(n+2)/2)j}\right]$$
$$var(\bar{Y}_{dmrss,e)} = \frac{1}{n^2}\left[\sum_{i=1}^{n/2}\sigma^2_{(i:n/2)} + \sum_{i=n/2+1}^{n}\sigma^2_{(i:(n+2)/2)}\right] \quad (9)$$

For simplicity, denote the mean of DMRSS by $\bar{Y}_{dmrssj} = (1/n)\sum_{i=1}^{n} y_{(i:n/2)j}$ and assume that the process mean is either known or can be estimated from the preliminary in-control process data and that σ^2 is known and constant. Following the setting in Section 2, we define the Shewhart and EWMA control limits based on DMRSS samples by

$$\text{SCL}_{dmrss} = k\sigma_{\bar{Y}dmrss}$$
$$\text{ECL}_{dmrss} = L\sigma_{\bar{Y}dmrss}\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2j}]} \quad (10)$$

where $\sigma_{\bar{Y}dmrss} = \sqrt{(1/n^2)\sum_{i=1}^{n}\sigma^2_{(i:n/2)}}$. Here, $\sigma^2_{(i:n/2)}$ is the ith variance for the DMRSS data which is defined by $\sigma^2_{(i:n/2)} = E\left[y_{(i:n/2)} - E\left(y_{(i:n/2)}\right)\right]^2$ and can be obtained by simulation or numerical integration. The DMRSS based CS-EWMA control chart gives a signal when the mean statistic $\bar{Y}_{drrssj}$ exceeds SCL_{dmrss} or the DMRSS EWMA statistic $S_{(j:n)} = \lambda\bar{Y}_{dmrssj} + (1-\lambda)S_{(j-1:n)}$ exceeds ECL_{dmrss}.

4. Performance measure

The statistical performance of a control chart is often measured by the run length (RL) properties. These include the average of the RL (ARL) which represents the average number of samples plotted on a control chart before an out-of-control signal is observed [2]. There is also standard deviation of the RL (SDRL), median of the RL (MRL) and the percentiles of the RL. In this Section, the RL properties of the proposed DMRSS based CS-EWMA control charts are computed numerically using Monte Carlo simulation through an algorithm developed in FORTRAN.

Suppose that the in-control process follows a normal distribution with mean $\mu_0 = 0$ and variance $\sigma_0^2 = 1$, without loss of generality. Define the shift in the process mean by $\delta = \sqrt{n}|\mu_{\text{out}} - \mu_0|/\sigma_0$, where μ_{out} is the out-of-control mean. Using $\lambda = 0.05, 0.1, 0.25, 0.5$ and 0.75 with different combinations of k and L, simulations were carried out based on $50{,}000$ iterations using a subgroup size of $n = 5$. The resulting properties of RL distribution for the proposed CS-EWMA charts are tabulated in Tables 1 – 4.

Table 1. ARL values for the CS-EWMA scheme based on DMRSS with $k = 3.25$ $(n = 5)$

δ	$\lambda = 0.05$	0.10	0.25	0.50	0.75
	$L = 2.718$	2.890	3.035	3.059	3.018
0.0	370.40	370.00	370.26	370.52	370.00
0.1	47.16	59.63	96.58	145.05	184.59
0.2	14.93	16.64	23.42	39.76	61.47
0.3	7.81	8.42	10.04	15.07	23.61
0.4	5.08	5.42	5.83	7.35	10.81
0.5	3.65	3.88	3.98	4.52	5.92
0.6	2.85	2.95	3.05	3.19	3.75
0.7	2.32	2.40	2.41	2.45	2.64
0.8	1.95	1.97	1.96	2.00	2.01
0.9	1.66	1.67	1.69	1.66	1.65
1.0	1.45	1.46	1.45	1.45	1.42
1.1	1.29	1.30	1.29	1.30	1.26
1.2	1.18	1.18	1.18	1.18	1.14
1.3	1.10	1.10	1.10	1.10	1.08
1.4	1.05	1.05	1.05	1.05	1.03
1.5	1.02	1.02	1.03	1.02	1.02

Based on the results in Tables 1–4, we summarize our findings for the proposed DMRSS based CS-EWMA control charts as follows:

1. The proposed scheme efficiently detect different size of changes in process mean level, ranging from small to large shifts (cf. Tables 1–4).
2. There is no significant difference between the in-control ($\delta = 0$) SDRL and ARL values (cf. Tables 1 and 2).

Table 2. SDRL values for the CS-EWMA scheme based on DMRSS with $k = 3.25$ $(n = 5)$

δ	$\lambda = 0.05$	0.10	0.25	0.50	0.75
	$L = 2.718$	2.890	3.035	3.059	3.018
0.0	369.43	369.53	370.75	367.55	370.60
0.1	37.00	51.54	93.99	141.93	183.30
0.2	9.35	11.29	19.81	38.21	60.05
0.3	4.28	4.71	6.87	13.12	22.34
0.4	2.54	2.69	3.33	5.55	9.61
0.5	1.67	1.80	2.05	2.90	4.89
0.6	1.24	1.32	1.41	1.82	2.66
0.7	0.99	1.04	1.09	1.21	1.66
0.8	0.81	0.86	0.86	0.92	1.11
0.9	0.68	0.72	0.73	0.71	0.83
1.0	0.58	0.60	0.59	0.59	0.64
1.1	0.48	0.50	0.49	0.49	0.50
1.2	0.39	0.40	0.40	0.40	0.37
1.3	0.30	0.30	0.30	0.30	0.28
1.4	0.22	0.22	0.22	0.22	0.17
1.5	0.15	0.14	0.16	0.15	0.14

Table 3. MRL values for the CS-EWMA scheme based on DMRSS with $k = 3.25$ $(n = 5)$

δ	$\lambda = 0.05$	0.10	0.25	0.50	0.75
	$L = 2.718$	2.890	3.035	3.059	3.018
0.0	254	256	254	258	253
0.1	38	45	68	102	129
0.2	13	14	18	28	42
0.3	7	8	8	11	17
0.4	5	5	5	6	8
0.5	3	4	4	4	4
0.6	3	3	3	3	3
0.7	2	2	2	2	2
0.8	2	2	2	2	2
0.9	2	2	2	2	1
1.0	1	1	1	1	1
1.1	1	1	1	1	1
1.2	1	1	1	1	1
1.3	1	1	1	1	1
1.4	1	1	1	1	1
1.5	1	1	1	1	1

Table 4. Percentile points for CS-EWMA scheme based on DMRSS with $k = 3.25$ $(n = 5)$

δ	$\lambda = 0.05$		0.10		0.25		0.50		0.75	
	P_{10}	P_{90}	P_{10}	P_{90}	P_{10}	P_{90}	P_{10}	P_{90}	P_{10}	P_{90}
0.0	38	85	40	8	41	8	40	85	40	85
0.1	11	95	12	1	14	2	17	33	20	42
0.2	5	28	5	3	5	4	6	90	8	14
0.3	3	14	3	1	3	1	3	32	4	53
0.4	2	8	2	9	2	1	2	15	2	23
0.5	2	6	2	6	2	7	2	8	2	12
0.6	1	4	1	5	1	5	1	5	1	7
0.7	1	4	1	4	1	4	1	4	1	5
0.8	1	3	1	3	1	3	1	3	1	3
0.9	1	3	1	3	1	3	1	3	1	3
1.0	1	2	1	2	1	2	1	2	1	2
1.1	1	2	1	2	1	2	1	2	1	2
1.2	1	2	1	2	1	2	1	2	1	2
1.3	1	1	1	1	1	1	1	1	1	1
1.4	1	1	1	1	1	1	1	1	1	1
1.5	1	1	1	1	1	1	1	1	1	1

3. The out-of-control ($\delta > 0$) ARL, SDRL and MRL values of the proposed scheme decrease rapidly for a given value of λ (cf. Tables 1–3).
4. The 10^{th} and 90^{th} percentile points of the RL distribution shows that the proposed scheme has positively skewed RL distributions (cf. Table 4).
5. The smaller the value of λ, the more gain in the performance of the proposed CS-EWMA control chart (cf. Tables 1 – 4).

5. Performance comparison

The statistical performance of the proposed DMRSS based CS-EWMA control chart is compared to some existing schemes for monitoring process mean level. Assuming that the underlying distribution is normal with a constant process variance, we used a subgroup size of $n = 5$ and set the in-control ARL value at 500 for a fair comparison. But instead of relying only on ARL values which measure the effectiveness of a control chart at some particular points [19], we use the Average Extra Quadratic Loss (AEQL), the Average Ratio of ARLs (ARARL) and Performance Comparison Index (PCI) to measure the overall effectiveness of the proposed scheme [19], [20], [21]. The smaller the AEQL and ARARL values of a control chart, the better the overall performance of the chart in detecting process changes.

The AEQL, ARARL and PCI are respectively defined as follows:

$$AEQL = \frac{1}{\delta_{max}-\delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \delta^2 ARL(\delta)\ \mathrm{d}\delta \quad (11)$$

$$ARARL = \frac{1}{\delta_{max}-\delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \frac{ARL(\delta)}{ARL_{best}(\delta)}\ \mathrm{d}\delta \quad (12)$$

$$PCI = AEQL/AEQL_{best} \quad (13)$$

where δ_{max} and δ_{min} are the upper and lower bound of the mean shift, respectively. $ARL(\delta)$ is the ARL value of a control chart at δ; $ARL_{best}(\delta)$ and $AEQL_{best}$ are generated by the best performing charts, which in this article correspond to the proposed CS-EWMA control chart. The other control charts used in this comparative study includes: the classical CS-EWMA chart, the RSS and MRSS based CS-EWMA control charts. Results obtained are presented in Tables 5 and 6, and ordered from left to right based on their detection ability.

5.1 Proposed verse the classical CS-EWMA

The classical CS-EWMA control chart introduced by [1] provided the much-needed protection against a different size of changes in a process mean. The scheme has archived significant improvement over the classical Shewhart and EWMA control charts. Comparison indicates that the proposed scheme uniformly performs better than the classical chart in terms of ARL. This point is equally supported by the overall performance, in terms of the AEQL, PCI and ARARL. In fact, the classical chart is the least scheme in this comparative study.

5.2 Proposed verse the RSS based CS-EWMA

The RSS based CS-EWMA control chart suggested by [7] is an improvement over the classical chart. The ARL values show that scheme is more effective than the classical CS-EWMA chart but has larger out-of-control ARL values than the proposed. Comparison of the AEQL and ARARL values also indicates that the RSS scheme is better than the classical but less efficient than the proposed control chart (cf. Tables 5 and 6).

Table 5. ARL comparison among the CS-EWMA control charts ($n = 5,\ \mathrm{ARL}_0 = 500$) when $\lambda = 0.10$

	Classical CS-EWMA	CS-EWMA RSS	CS-EWMA MRSS	CS-EWMA DMRSS
δ	$L = 3.021$ $k = 3.310$	$L = 3.052$ $k = 3.310$	$L = 3.021$ $k = 3.310$	$L = 2.981$ $k = 3.350$
0.0	500.36	500.78	500.33	500.53
0.1	358.64	239.14	200.10	68.22
0.2	184.22	81.50	62.91	18.04
0.3	95.73	36.65	28.22	8.98
0.4	55.09	20.81	16.47	5.65
0.5	35.46	13.81	11.12	4.06
0.6	24.78	10.15	8.18	3.11
0.7	18.43	7.82	6.40	2.49
0.8	14.53	6.31	5.23	2.06
0.9	11.78	5.26	4.36	1.75
1.0	9.92	4.48	3.75	1.51
1.1	8.47	3.89	3.23	1.33
1.2	7.36	3.40	2.85	1.21
1.3	6.49	3.04	2.54	1.11
1.4	5.74	2.73	2.29	1.06
1.5	5.16	2.46	2.05	1.03
AEQL	8.854	3.924	3.218	1.280
PCI	6.919	3.067	2.514	1.000
ARARL	7.133	3.157	2.575	1.000

Table 6. ARL comparison among the CS-EWMA control charts ($n = 5,\ \mathrm{ARL}_0 = 500$) when $\lambda = 0.25$

	Classical CS-EWMA	CS-EWMA RSS	CS-EWMA MRSS	CS-EWMA DMRSS
δ	$L = 3.158$ $k = 3.310$	$L = 3.187$ $k = 3.310$	$L = 3.158$ $k = 3.310$	$L = 3.123$ $k = 3.350$
0.0	500.15	500.15	500.24	500.13
0.1	416.18	322.20	286.25	115.31
0.2	269.87	139.43	108.13	26.57
0.3	159.62	62.25	46.31	10.77
0.4	95.30	32.21	23.50	6.14
0.5	59.49	19.03	14.26	4.20
0.6	39.52	12.57	9.68	3.13
0.7	27.40	9.10	7.14	2.52
0.8	20.00	7.04	5.60	2.06
0.9	15.45	5.65	4.56	1.73
1.0	12.23	4.69	3.83	1.51
1.1	10.06	4.02	3.30	1.33
1.2	8.41	3.48	2.88	1.20
1.3	7.19	3.07	2.54	1.12
1.4	6.29	2.74	2.28	1.06
1.5	5.55	2.46	2.06	1.03
AEQL	11.977	4.684	3.731	1.353
PCI	8.854	3.463	2.759	1.000
ARARL	9.238	3.595	2.844	1.000

5.3 Proposed verse the MRSS based CS-EWMA

To offer better protection against range of mean shifts, [7] studied the MRSS based CS-EWMA control chart as enhancements over the RSS counterpart. This scheme is outlier-resistant and prone to fewer ranking errors as it does not require accurate measurement of every observation. The out-of-control ARL values of the MRSS based CS-EWMA control chart indicates that the scheme is uniformly better than the classical and RSS based CS-EWMA charts but much poorer than the proposed. The overall performance in terms of AEQL, PCI and ARARL has similar conclusions (cf. Tables 5 and 6).

6. Application example

This section presents a practical example to illustrate the application of our proposed CS-EWMA control chart using real dataset based on an example [8] on fill volume of a soft drink bottle. The original dataset was obtained from a production line of the Pepsi-Cola production company, Al-Khobar, Saudi Arabia. Using the approach of [22], we re-sample the original data to obtain new SRS, RSS, MRSS and DMRSS dataset with perfect ranking. Figure 1 displays the averages of the thirty six re-sampled data points, each of subgroup size three. To test how sensitive is the proposed scheme to shifts in process mean level, each of the last 10 data points was contaminated by adding 0.2 units to take the process out-of-control.

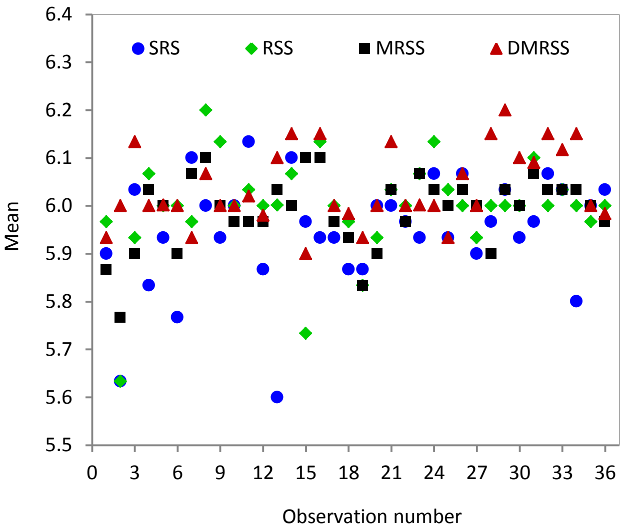

Figure 1. Averages of the re-sampled dataset collected using SRS, RSS, MRSS and DMRSS

The CS-EWMA charts were constructed based on the computed mean and the standard deviation for the thirty-six samples. The charts parameters are based on the setting in Section 5 when $\lambda = 0.1$ using an in-control ARL of 500. Figures 2–5 show the graphical display of the classical CS-EWMA chart, RSS based CS-EWMA chart, MRSS based CS-EWMA chart and the proposed scheme.

Based on real data used for this application example, we observe that the classical CS-EWMA chart is not doing so well as it gives only five out-of-control data points (cf. Figure 2). However, all the ranked data based CS-EWMA charts are not doing badly at all with the proposed scheme dominating the other two (cf. Figures 3-5). The Shewhart feature of the RSS chart appears to be struggling as it could signal only one point compared to the five points of MRSS and seven points of DMRSS. Because the shift is small, the EWMA components of the MRSS and DMRSS based CS-EWMA chart are giving the right out-of-control points, but the RSS is one point less.

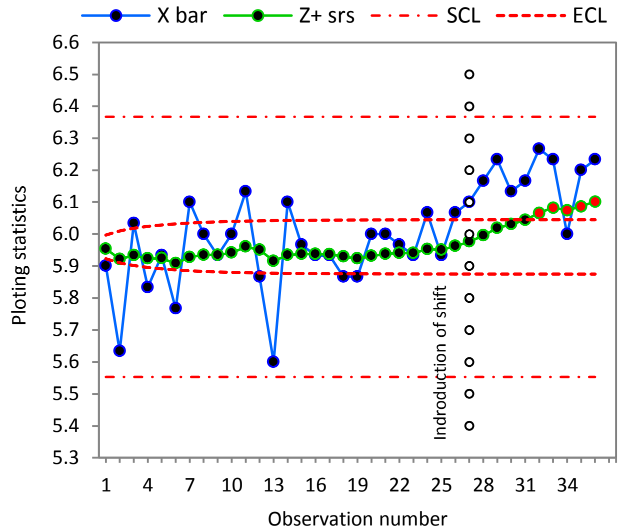

Figure 2. Classical CS-EWMA control chart using real dataset

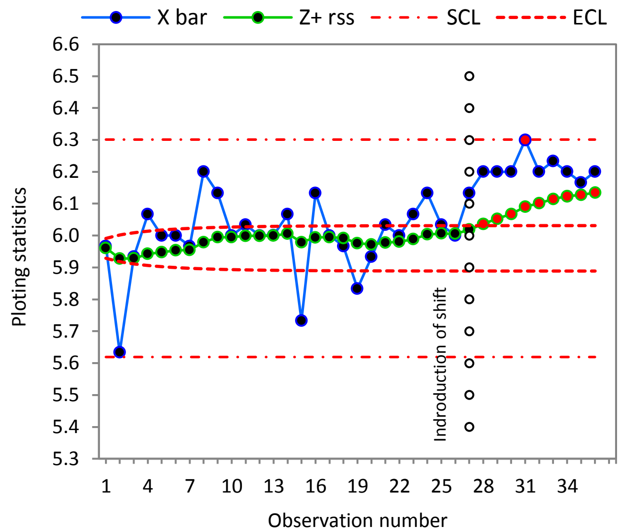

Figure 3. RSS based CS-EWMA control chart using real dataset

7. Conclusions and recommendations

In this study, we have proposed a new CS-EWMA control chart based on DMRSS as an improvement over those based on RSS and MRSS. The RL properties of the proposed scheme are evaluated and found to be substantially more effective in the detection of different sizes of shifts in the process mean than the classical, RSS and MRSS based CS-EWMA charts without increasing the false alarm rate. The scope of this study may be extended to other control charting structures.

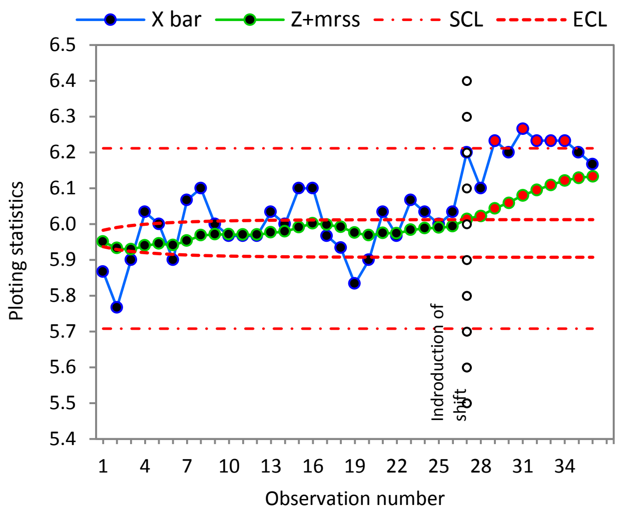

Figure 4. MRSS based CS-EWMA control chart using real dataset

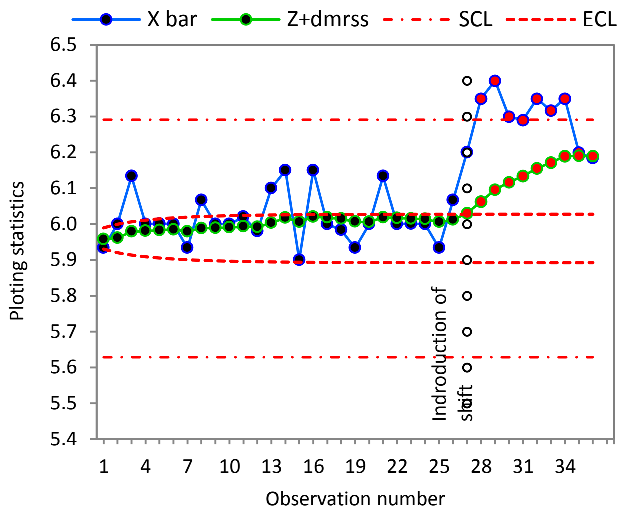

Figure 5. Proposed DMRSS based CS-EWMA control chart using real dataset

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